THE TRANSIT

Navigation Satellite System

By Thomas A. Stansell

STATUS
THEORY
PERFORMANCE
APPLICATIONS

Magnavox
ADVANCED PRODUCTS AND SYSTEMS COMPANY

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CHAPTER 1
INTRODUCTION AND SUMMARY

The purpose of this document is to provide an in-depth review of Transit, the Navy Navigation Satellite System, from the user's point of view. After a brief system description, a spectrum of diverse applications is described, ranging from the navigation of fishing boats to guiding submarines. Next, the Transit system status and its vitality are discussed. It becomes clear that the system is exceptionally reliable and trustworthy, that the use of and the investment in Transit equipment is growing at a remarkable rate, and that the basic system is about to be improved by the addition of a new generation of NOVA satellites. From these indications and the navigation planning initiatives described in Reference 12, this author concludes that Transit will continue to provide a valuable service until at least 1995, after which phase-over to the Global Positioning System is expected to be complete.

The second half of this document is devoted to a technical description of the position fix process and of the factors which influence accuracy. The satellite signal structure, the meaning of the navigation message, and the interpretation of Doppler measurements are covered in detail, followed by an overview of the fix calculation process. Finally, a thorough review of the system accuracy potential and of the factors which determine accuracy performance is given.

The Transit system grew out of the confluence of a vital need with newly available technology. (See Reference 17 for a complete review.) The need was to have accurate position updates for the inertial navigation equipment aboard Polaris submarines. The new space age technology came into being because of Sputnik I, which was launched on October 4, 1957. Drs. William H. Guier and George C. Weiffenbach of the Applied Physics Laboratory of Johns Hopkins University (APL) were intrigued by the substantial Doppler frequency shift of radio signals from this first artificial earth satellite. Their interest led to development of algorithms for determining the entire satellite orbit with careful Doppler Measurements from a single ground tracking station. Based on this success, Drs. Frank T.

McClure and Richard B. Kershner, also of APL, suggested that the process could be inverted, i.e., a navigator's position could be determined with Doppler measurements from a satellite with an accurately known orbit.

Because of the confluence of need with available technology, development of Transit was funded in December 1958. Under the leadership of Dr. Kershner, three major tasks were addressed: development of appropriate spacecraft, modeling of the earth's gravity field to permit accurate determination of satellite orbits, and development of user equipment to deliver the navigation results. Transit became operational in January of 1964, and it was released for commercial use in July of 1967. The user population has grown rapidly since that date, as detailed in Sections 4.1 and 4.4 of this document, and today commercial users far outnumber government or military users. Of considerable interest is the amazing diversity of applications which will be described in Chapter 3.

CHAPTER 2
TRANSIT SYSTEM DESCRIPTION

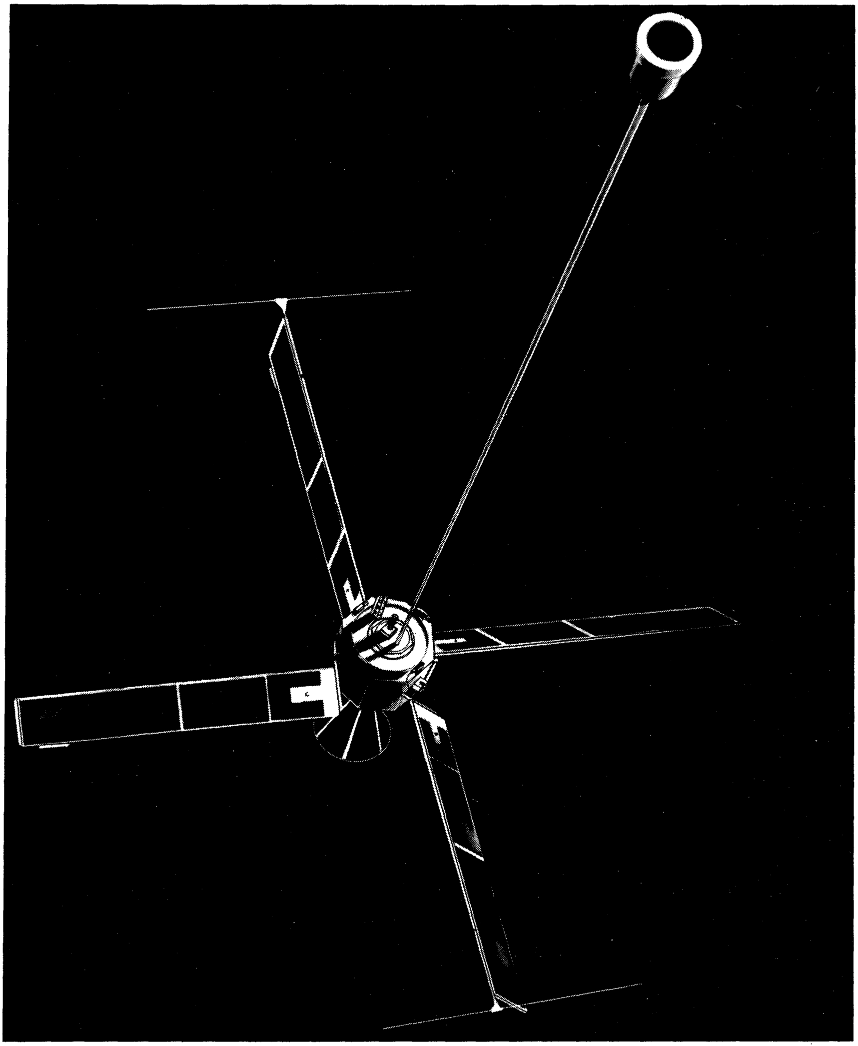


Figure 1. Physical Configuration of Transit Satellites

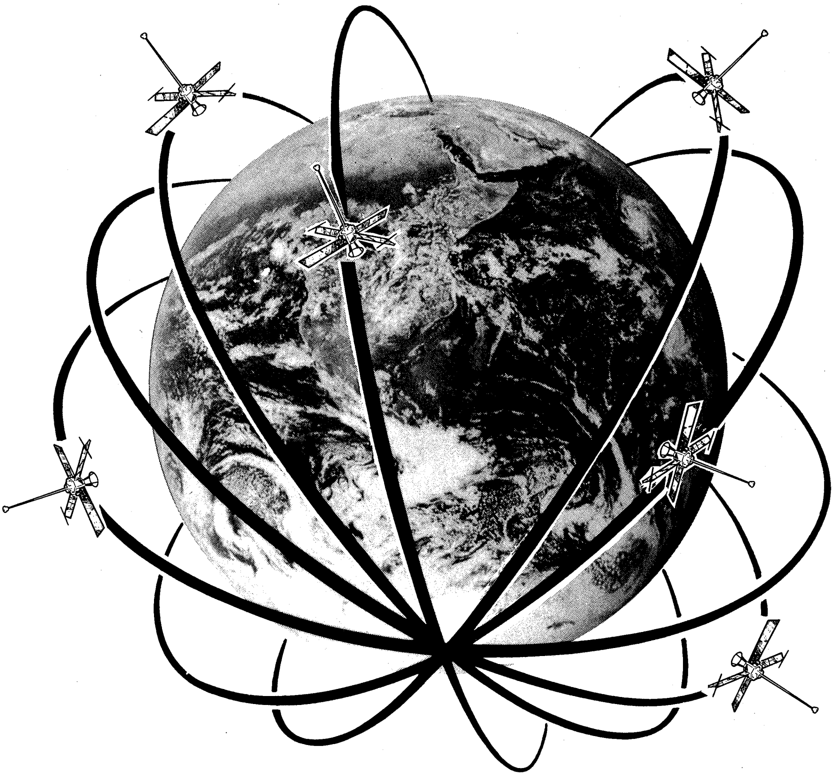


Figure 2. Transit Satellites Form a "Birdcage" of Circular, Polar Orbits About 1075 km Above the Earth

This chapter is a very brief description of the Transit system, permitting the reader to move quickly into a review of system applications. More detailed system descriptions will be provided in later chapters of this document.

The Applied Physics Laboratory of Johns Hopkins University (APL) has played the central role in development of Transit. The original idea was conceived there, most of the actual development was performed there, and APL continues to provide technical support in maintaining and improving the system.

At this time there are five operational Transit satellites in orbit. Figure 1 illustrates their physical configuration: four panels of solar cells charge the internal batteries, and signals are transmitted to the earth by the "lampshade" antenna, which always points downward because of the gravity gradient stabilization boom. An elongated object in orbit naturally tries to align with the earth's gravity gradient. Magnetic hysteresis rods along the solar panels damp out the tendency to sway back and forth by interaction with the earth's magnetic field; that is, mechanical energy is converted to heat through magnetic hysteresis.

As illustrated by Figure 2, the satellites are in circular, polar orbits, about 1,075 kilometers high, circling the earth every 107 minutes. This constellation of orbits forms a "birdcage" within which the earth rotates, carrying us past each orbit in turn. Whenever a satellite passes above the horizon, we have the opportunity to obtain a position fix. The average time interval between fixes with the existing 5 satellites varies from about 35 to 100 minutes depending on latitude, as shown in Figure 3. Sections 4.3 and 4.7 describe plans for additional satellites which will improve the time interval statistics.

Transit is operated by the Navy Astronautics Group headquartered at Point Mugu, California, with tracking stations located at Prospect Harbor, Maine; Rosemount, Minnesota; and Wahiawa, Hawaii. As illustrated by Figure 4, each time a Transit satellite passes within line of sight of a tracking station, it receives the 150 and 400 MHz signals transmitted by the satellite, measures the Doppler frequency shift caused by the satellite's motion, and records the Doppler frequency as a function of time. The Doppler data are then sent to the Point Mugu computing center where they are used to determine each satellite's orbit and to project each orbit many hours into the future.

The computing center forms a navigation message from the predicted orbit, which is provided to the injection stations at Point Mugu and at Rosemount. At the next opportunity, one of the injection stations transmits the navigation message to the appropriate satellite. Each satellite receives a new message about every 12 hours, although the memory capacity is 16 hours.

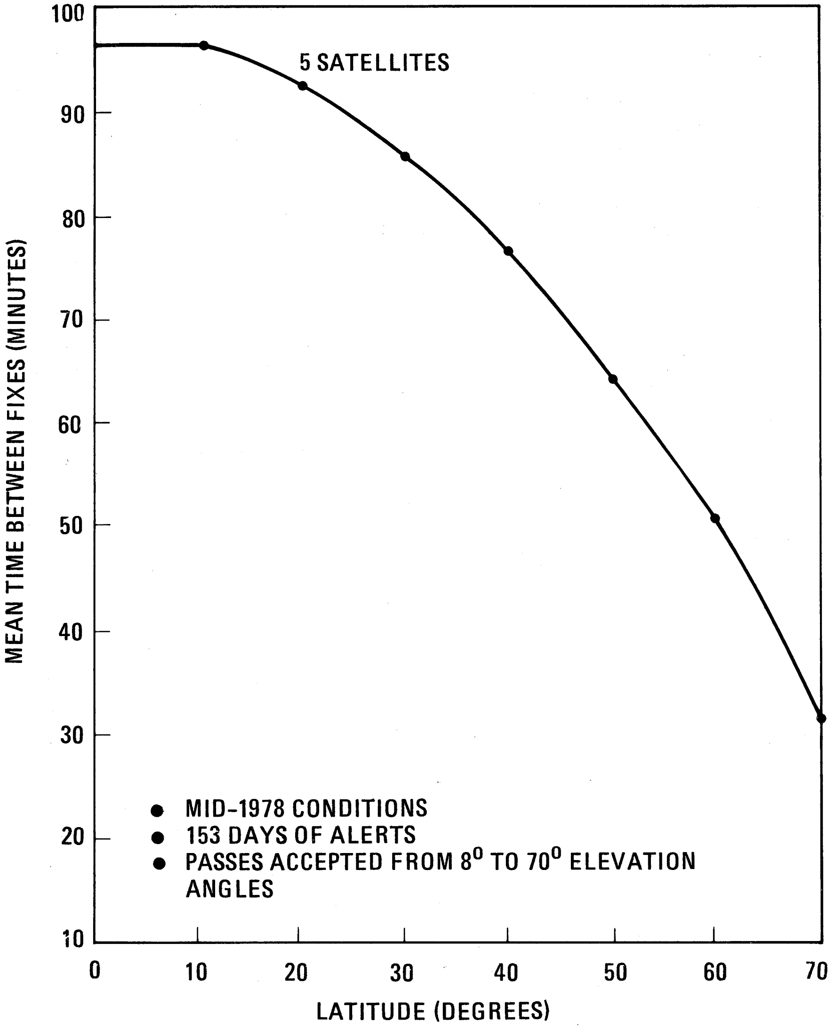


Figure 3. Mean Time Between Position Fixes as a Function of Latitude with the 5 Transit Satellites Operational in mid-1978

Unlike earth-based radiolocation systems which determine position by nearly simultaneous measurements on signals from several fixed transmitters, Transit measurements are with respect to sequential positions of the satellite as it passes, as illustrated by Figure 5.

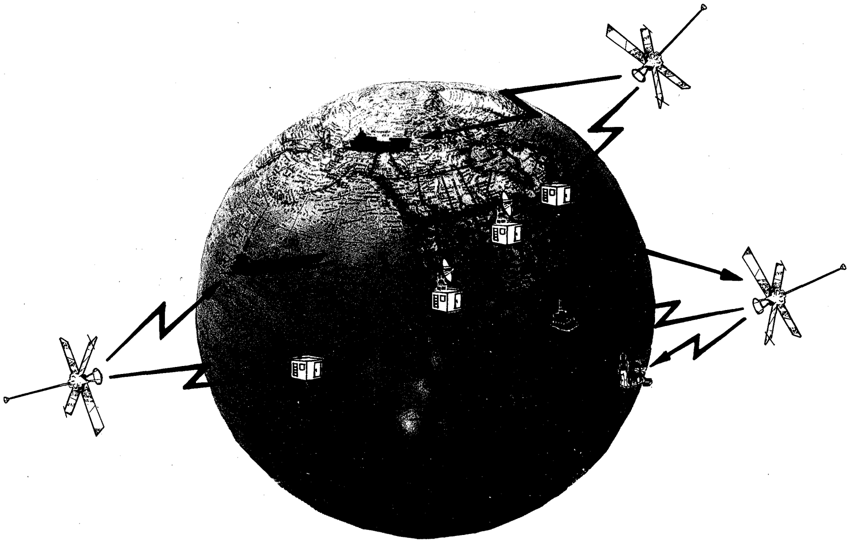


Figure 4. Schematic Overview of the Transit Navigation Satellite System

This process requires from 10 to 16 minutes, during which time the satellite travels 4,400 to 7,000 kilometers, providing an excellent baseline.

Because Transit measurements are not instantaneous, motion of the vessel during the satellite pass must be considered in the fix calculations. Also, because the satellites are in constant motion relative to the earth, simple charts with lines of position are impossible to generate. Instead, each satellite transmits a message which permits its position to be calculated quite accurately as a function of time. By combining the calculated satellite positions, range difference measurements between these positions (Doppler counts), and information regarding motion of the vessel, an accurate position fix can be obtained. Because the calculations are both complex and extensive, a small digital computer is required.

Transit is the only navigation aid with total worldwide availability at this time. It is not affected by weather conditions, and position fixes have an accuracy competitive with short range radiolocation systems. Each satellite is a self-contained navigation beacon which

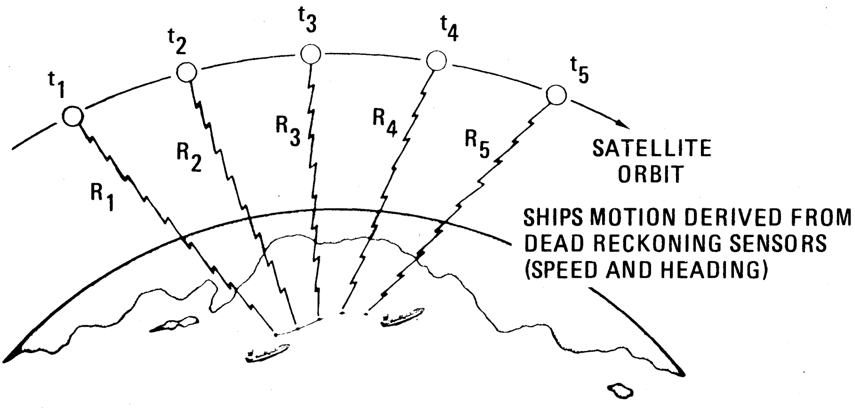


Figure 5. Geometry of a Satellite Pass

transmits two very stable frequencies (150 and 400 MHz), timing marks, and a navigation message. By receiving these signals during a single pass, the system user can calculate an accurate position fix.

There are two principal components of error in a Transit position fix. First is the inherent system error, and second is error introduced by unknown ship's motion during the satellite pass. The inherent system error can be measured by operating a Transit set at a fixed location and observing the scatter of navigation results. Figure 6 is a plot of such data from a dual-channel Transit receiver showing a radial scatter of 32 meters rms. Dual-channel results typically fall in the range of 27 to 37 meters rms. Less expensive single-channel receivers, which do not measure and remove ionospheric refraction errors, typically achieve results in the range of 80 to 100 meters rms.

The second source of position fix error is introduced by unknown motion during the satellite pass. The exact error is a complex function of satellite pass geometry and direction of the velocity error, as explained in Chapter 6 of this document, but a reasonable rule is

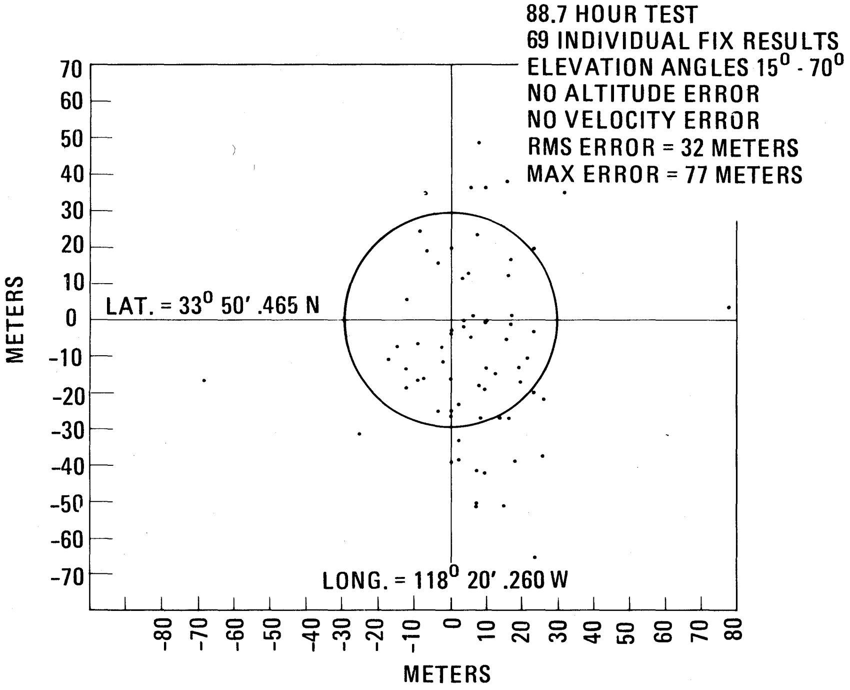
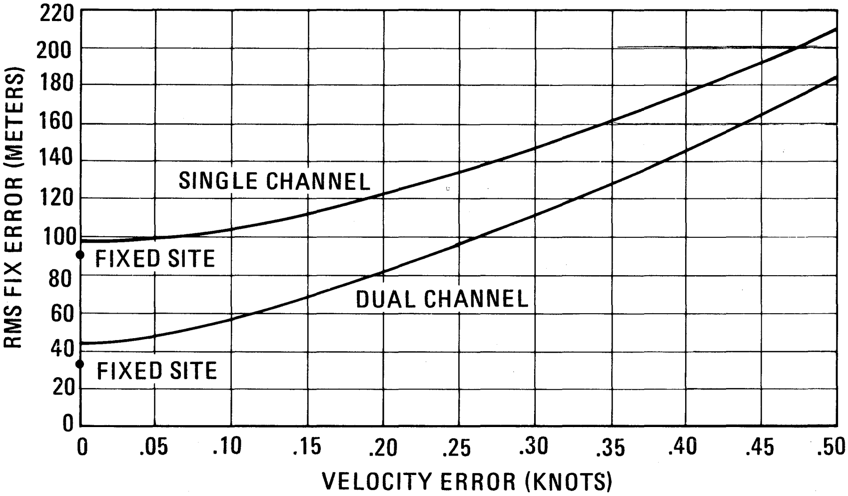


Figure 6. Typical Dual-Channel Satellite Position Fix Results

that 0.2 nautical mile (370 meters) of position error will result from each knot of unknown ship's velocity. Figure 7 is a plot of approximate position fix error as a function of unknown velocity magnitude for dual-channel and for single-channel Transit receivers. The effects of typical altitude errors and ship's pitch and roll have been included in this curve as well.

Figure 8 illustrates the preferred mode of operation for a moving navigator. Between satellite fixes the computer automatically dead reckons based on inputs of speed and heading. The dead reckoning process also is used to describe ship's motion during each satellite pass. After the position fix has been computed, latitude and longitude adjustments are applied, thus correcting for the accumulated dead reckoning error.



NOTES: 1. MAXIMUM SINGLE-CHANNEL FIX ERROR CAN REACH 200 TO 500 METERS DUE TO IONOSPHERIC REFRACTION VERSUS ONLY 90 METERS OF MAXIMUM FIX ERROR FOR DUAL-CHANNEL RESULTS.

2. SOLVING FOR VELOCITY NORTH CAN LIMIT FIX ERROR TO THE RANGE OF 100-200 METERS WHEN VELOCITY IS POORLY KNOWN.

Figure 7. Approximate Satellite Position Fix Error as a Function of Unknown Velocity Magnitude

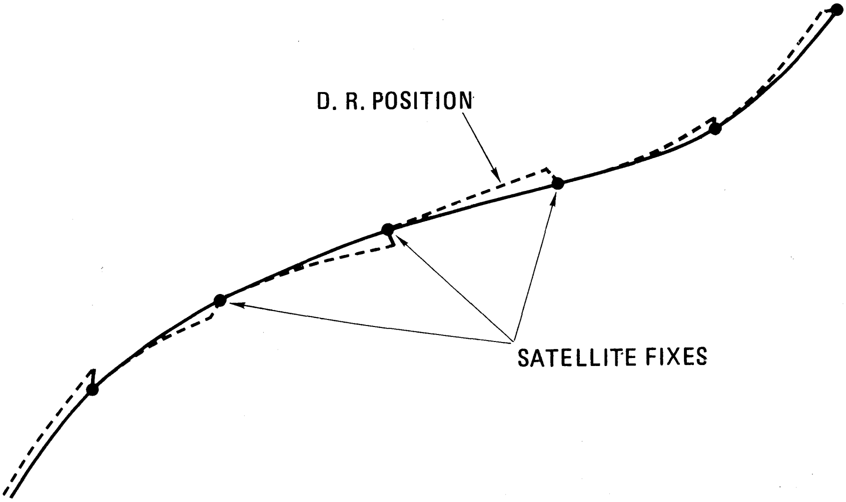


Figure 8. Dead Reckoning Error is Corrected by Each Satellite Position Fix Update

CHAPTER 3
TRANSIT APPLICATIONS

3.1 PRODUCT TRENDS

The Transit system provides a combination of capabilities which cannot be obtained with any other system today. These are:

- Total global coverage
- All weather operation
- Accuracy approaching that of short range radiolocation systems
- Independence from shore-based transmitters
- Unequaled dependability

As a result, there has been a steady and dramatic increase in both the number of applications and the types of equipment available. The range of applications is truly surprising. Transit equipment is used aboard and/or for:

- Land Survey
- Fishing boats
- Private yachts
- Commercial ships (tankers, freighters, etc.)
- Military surface ships
- Submarines
- Offshore drill rigs
- Oil exploration vessels
- Oceanographic research vessels
- Hydrographic survey vessels
- Drifting buoys

To match the growing user interest and to take better advantage of available technology, Transit user equipment has evolved dramati-

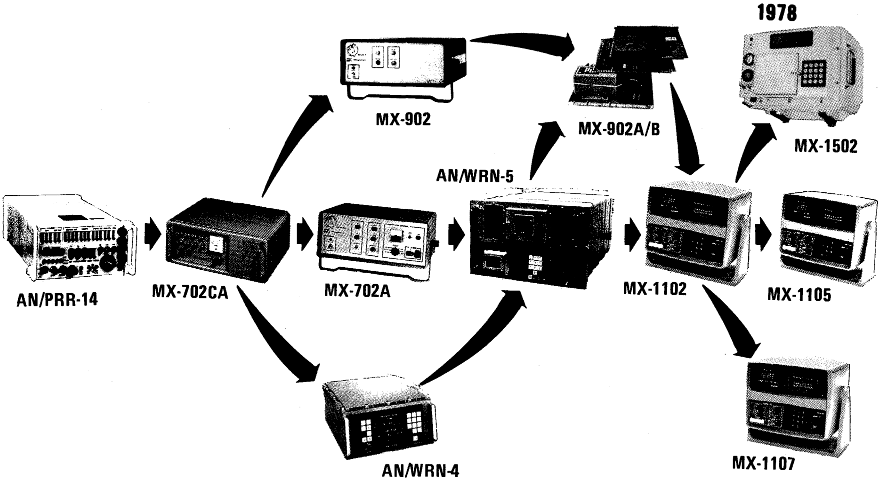


Figure 9. Evolution of Magnavox Transit Receiver Technology

cally since the early equipment designs of 1967. Figure 9 is one view of this evolutionary process, showing the many different types of Transit receivers developed since 1967 by just one company.

Figure 10 is another view of the equipment progress, showing the evolution of Magnavox single-channel satellite navigators from 1968 through 1976. In 1968 only dual-channel receivers were available, and a minicomputer occupied most of a rack. In 1971 a single-channel receiver was introduced and by then minicomputers were only 12 inches high. In 1973 the noisy, electromechanical Teletype was replaced by a quiet and compact video terminal with a cassette tape reader for loading the computer program. In 1975 new technology permitted the receiver to be implemented on a pair of circuit boards which fit within the computer. Also, minicomputers became smaller, permitting greater freedom in the shipboard installation.

The final step in Figure 10 is the first production satellite navigator based on microcomputer technology, the MX 1102. In addition to being smaller, less expensive, and far more reliable than its predecessors, this new type of navigator also has more functional capability.

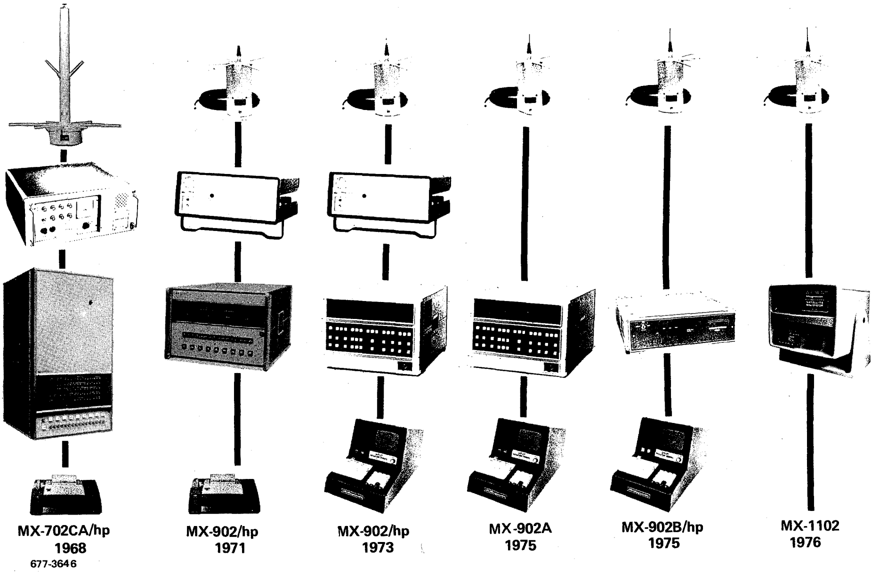


Figure 10. Evolution of Magnavox Single-Channel Satellite Navigation Equipment

For example, the MX 1102 not only tests itself thoroughly every two hours, but it will identify which module to replace if a failure does occur. Actual field results show a reliability of well over one year mean time between failure (MTBF). Thus, modern technology has lowered the cost and improved the capability of satellite navigation instruments.

3.2 GENERAL NAVIGATION

Because of availability of instruments like the MX 1102 shown in Figure 11, general navigation applications of Transit have dramatically increased in the last year or two. Such instruments provide a continuous display of latitude, longitude, and Greenwich mean time by continuously dead-reckoning between accurate Transit position fixes with automatic speed and heading inputs. In addition to the basic navigation functions, such systems determine and compensate for unknown set and drift, provide great circle or rhumb line range and bearing to any selected way point, determine the heading to steer to these way points, and in case of failure identify the faulty module.

Figure 11. Magnavox Satellite Navigator MX 1102

Typical applications include use aboard large fishing vessels. For example, when fishing for tuna in the southern hemisphere no other navigation aid provides the coverage or the dependable accuracy needed to assure success and to avoid fishing within 200-mile limits. Success is measured by which boat returns first with full coolers, and Transit navigation has measurably improved the rate of success.

Several large shipping companies in 1977 conducted competitive evaluations of various types of navigation equipment (Loran, Omega, and Transit, each from several manufacturers). Transit won each of these evaluations, and as a result entire commercial fleets are being equipped with Transit navigators. This trend is growing as the economic and safety advantages of dependably accurate worldwide navigation is proved over and over again. The availability of instruments with a low initial cost and with outstanding reliability records, so that life cycle support costs are minimized, also has spurred the interest of major fleet operators. The need for accurate, depend-

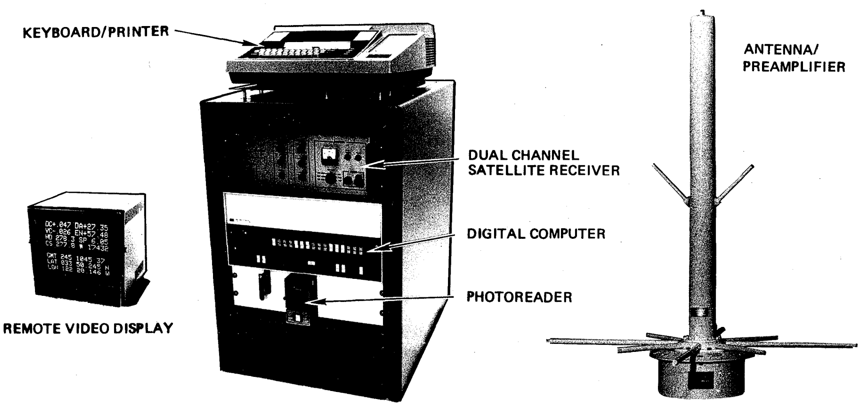


Figure 12. Typical Dual-Channel Equipment Used for Oceanographic Exploration

able, worldwide navigation is real. For example, oil tankers passing through the Straits of Malacca truly depend on these characteristics. Often a ship will time its arrival to obtain a satellite fix just before proceeding through such hazardous waters.

3.3 OCEANOGRAPHIC EXPLORATION

The first application of Transit navigation beyond its original military objectives was for oceanographic exploration. For the first time, mid-ocean scientific measurements could be tied to their geographic origin with high accuracy. The AN/WRN-4 equipment shown in Figure 9 and the equipment shown in Figure 12 are typical of the dual-channel Transit systems often used for oceanographic exploration.

In addition to the capabilities provided by commercial single-channel equipment, such as the MX 1102 of Figure 11, the dual-channel equipment gives high accuracy position fixes that are unaffected by variations in ionospheric refraction. In addition, it is typical for the system to provide a printed record of the dead-reckoned position at selected time intervals and of every satellite fix with appropriate quality indicators.

The equipment described above is now yielding to the advent of

Figure 13. Magnavox MX 1107 Dual-Channel Satellite Navigator and Printer

microcomputers. Figure 13 shows the Magnavox MX 1107 dual-channel satellite navigator with associated printer. This new instrument provides the same navigational accuracy capabilities as the much larger equipment shown in Figure 12.

3.4 GEOPHYSICAL SURVEY

3.4.1 Background

In 1967 when Transit was first released for civil use, there were two immediate positive responses. One was from the oceanographic exploration community, and the other was from the offshore oil exploration community. The oceanographers were among the first civil users, but their needs have remained fairly static since the early systems were acquired. In contrast, offshore oil exploration needs have continued to grow and to become more complex.

Prior to 1967 all offshore exploration was conducted with the aid of shore-based radiolocation systems such as Raydist, Hi-fix, etc. These systems work very well, but they have several serious problems.

- Usable range is limited, especially at night.
- The administrative and logistics costs of obtaining government approvals, transporting the equipment, installing and

surveying in the shore-based stations, and operating these stations in sometimes hostile environments are very high indeed.

- Most such systems require the counting of lane crossings, and the potential for lane slips is high. This forces expensive means for occasionally verifying or correcting the lane count.

When Transit was released, there were visions of accurate, worldwide, all-weather survey operations without the time and expense required to cope with shore-based radiolocation systems. Unfortunately, simply buying a Transit navigation system did not achieve these objectives.

3.4.2 The Need for Integration

Transit provides intermittent position fixes with an individual accuracy of 27 to 37 meters, but with an additional error of about 0.2 N.Mi. per knot of unknown velocity. Survey work requires the high accuracy, but continuously. Thus, the only way to provide continuous, accurate navigation independent of shore-based stations was to combine accurate velocity sensors with the Transit fix capability in an integrated system. The first such systems were relatively crude, but very capable systems quickly evolved. Figure 14 shows a typical integrated navigation system.

3.4.3 Doppler Sonar and Gyrocompass

The first system elements to be integrated were a Doppler sonar and a gyrocompass. The Doppler sonar transmits pulses of acoustic energy to the ocean floor and evaluates the signals reflected back. The Doppler frequency shift provides an accurate measure of ship's speed with respect to the bottom in the direction of each sonar beam. Three or four beams are used to determine both fore-aft and port-starboard components of total velocity. An additional requirement is knowledge of speed of sound in water near the sonar transducer. In most cases this can be determined to satisfactory accuracy by measuring water temperature, but if salinity is likely to change drastically, a velocimeter is required for best results.

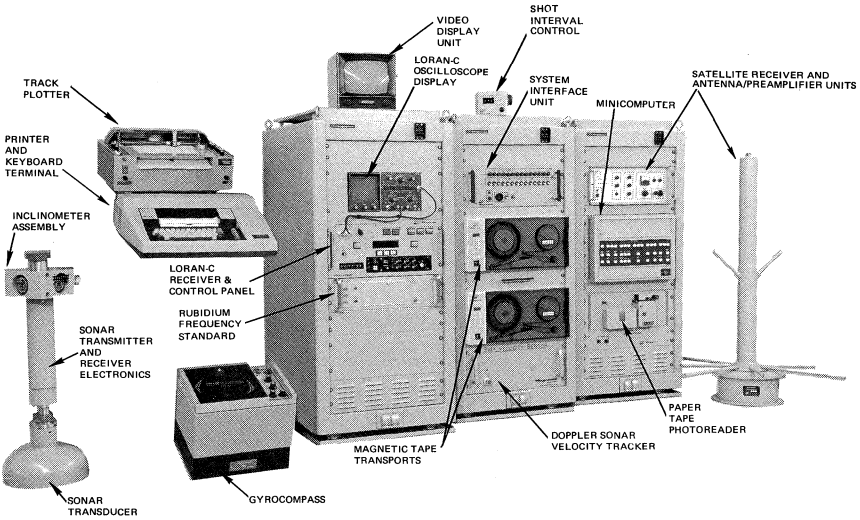


Figure 14. Typical Integrated Navigation System Components

Early Doppler sonars were limited to about 200 meters of water depth before they could no longer track the bottom and had to switch to a water tracking mode, which is much less accurate. The usual Doppler Sonar today will bottom track to 300 or 400 meters, there are models available which will reach 1,000 meters or more, and systems are being developed which promise bottom tracking to maximum ocean depths.

Gyrocompasses such as the Sperry Mk-227 or the Arma Brown MK-10 compass shown in Figure 14 have been used with good success. In both cases it is important to implement automatic computer torquing of the gyrocompass to compensate for latitude, velocity, and accelerations. Not only can the computer do a better job than would be possible with the usual manual control settings, but the automatic approach avoids a major error source — the human mistake.

Navigational accuracy is dependent on a number of factors, including complement of equipment, adequacy of calibration, water depth, and sea state. Figure 15 shows how position error grows with time

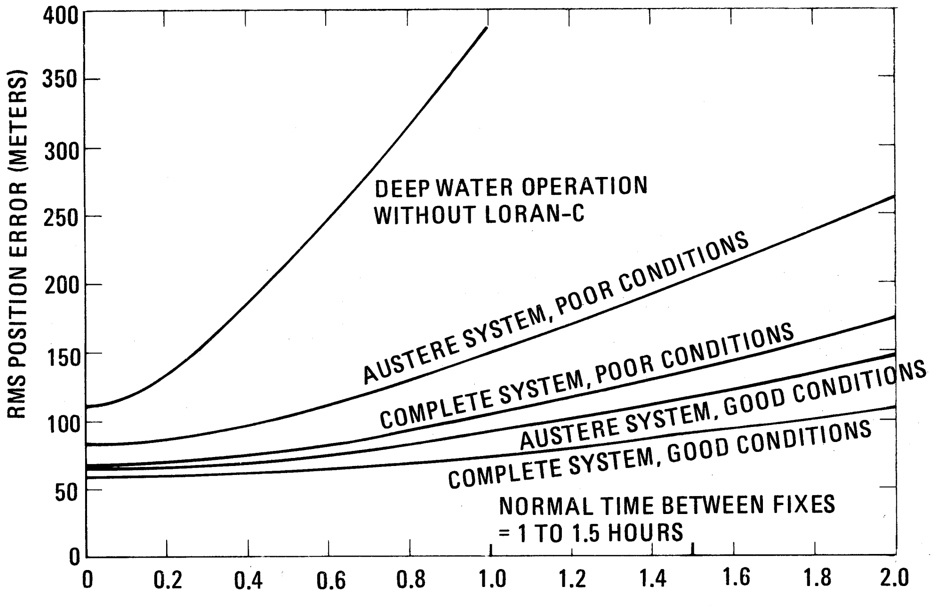


Figure 15. Integrated Sytem Error as a Function of Time Since the Last Satellite Fix Update

since the last satellite fix for a complete system and for an austere integrated system under both good and poor conditions. The figure also reveals how error increases much more rapidly when the sonar cannot reach bottom, unless a radio aid such as Loran-C is available.

3.4.4 Radio Navigation Aids

In very deep water or where it is not possible to install a Doppler sonar, some other source of velocity is needed. In many cases various radionavigation signals may be available already. For example, Figure 14 shows Loran-C components as part of the integrated system. Loran-C alone would not have sufficient accuracy because of secondary phase errors and often because of poor "crossing angles". However, by integrating Loran-C with other system elements, excellent accuracy can be obtained. Satellite fixes provide a precise geographic position reference and provide calibration of local Loran-C secondary phase errors. By having a gyrocompass and a Doppler sonar, even if in the water track mode, ship's maneuvers can be determined accurately. This permits the Loran-C readings to be filtered heavily, thus removing most of the random noise. In effect Loran-C is used to correct for the effects of unknown set and drift. Furthermore, because satellite fixes are available to provide an accurate position reference, it is possible to use Loran-C in the delta-range measurement mode with respect to a rubidium or cesium clock. Because delta-range measurements can be made on each Loran-C signal independently, useful information can be obtained with only one or two Loran-C signals, which greatly expands the area of accurate coverage and reduces the problem of poor crossing angles.

The same concepts can be used with a wide variety of radionavigation systems. When integrating with short range, high accuracy systems, a speed sensor is not needed, and the satellite fix capability is used to verify and resolve lane counts. Systems have been implemented with such radionavigation aids as: Decca Navigator, Hi Fix, Raydist, Toran, Argo, Miniranger, Trisponder, and others. Each one has its advantages, so flexible hardware and software is provided for rapid configuration with any appropriate radio navigation sensor.

3.4.5 Acoustic Transponders

One of the most sophisticated versions of an integrated system employs acoustic transponders (see References 14 and 15). The ship is equipped with an interrogater/receiver set. Every few seconds the interrogator sends out an acoustic pulse at a specific frequency. Transponders which have been placed on the bottom and are within range receive the interrogate pulse and respond by sending a pulse of their own at an individual frequency. The receiver on the ship picks up and identifies these replies and measures the total round trip delay. Such measurements, scaled with an appropriate estimate of speed of sound in water, define the range to each transponder. If the position of each transponder is known accurately, then a navigational accuracy of 2 to 10 meters can be achieved typically over an area of 3 to 10 square kilometers with only a few bottom transponders. Such systems are being used for site surveys and for precise drill rig positioning during the final approach. Although expensive, it may be the only way to achieve the required accuracy for 3-dimensional seismic surveys as well.

In the previous paragraph there was a big "if"; if the position of each transponder is known accurately. This is the difficult part. Special software has been developed to determine the transponder positions with great accuracy and in a minimum of time. The first step is to collect transponder range readings while following a specific pattern around each transponder location. Because the equations must be solved iteratively, these data are recorded in memory and used over and over until the total solution converges and the relative position of each transponder is known accurately. This technique saves time by requiring the ship to traverse the area only once; the computer does all the work after that.

Once the relative transponder positions are known, it is often necessary to determine their true latitude and longitude positions as well. This is achieved with the aid of multiple satellite position fixes. Motion of the ship relative to the transponder net can be determined accurately, but the position (translation) and azimuth (rotation) of the net are unknown. Again, an iterative solution is used in which each satellite fix improves knowledge of the net position and azimuth. As knowledge of net azimuth improves, the measure of ship's motion becomes more accurate. Such iterations are best done with all raw satellite and transponder data recorded on magnetic tape, and the technique has proved to be extremely effective and accurate.

3.4.6 Integrated Navigation System Functions

A wide variety of integrated navigation systems have been developed and deployed to aid offshore exploration. However, navigation is just one of the three major functions of an integrated system. The other two are survey control and data logging.

The system helps control the survey, for example, by firing seismic shots at defined increments of time or of distance traveled. In some installations the system actually controls steering of the vessel along the desired survey path.

Data logging is the third necessary ingredient. Unless the position at which the geophysical data were acquired is recorded, the data are worthless. Therefore, data logging must be extremely reliable result in 3-dimensions (latitude, longitude, and altitude). The geodetic reference for such a position determination is provided by the satellite system itself.

If a reference station can be occupied within several hundred kilometers of a survey site, a technique called translocation can produce greater accuracy in less time. To implement the translocation technique, two or more satellite receivers are used, one at the reference site and the other(s) at the survey site(s). By tracking the same satellite passes, improved accuracy is achieved because the computer solves for differential position between the two points, which is not affected by common error sources.

The U.S. Government conducts many surveys with Transit satellites. The instrument normally used is the AN/PRR-14 Geoceiver shown in Figure 16. For example, adjustment of the North American Datum which is now underway depends heavily on results obtained with the Geoceiver at many survey points across all of North America. In reducing Geoceiver data, the Government has an advantage not available to the private user. This is postcomputation of each satellite orbit based on data from tracking stations taken concurrently with the survey. The result is a "precise ephemeris" orbit definition.

3.5.3 Equipment

Several different types of portable survey equipment have been developed. The original, which is still in wide use, is the AN/PRR-14 Geoceiver shown in Figure 16. On the left is the four-frequency receiver (which tracks both Transit and GEOS satellites), at the center is the antenna and preamplifier on a tripod, and on the right is the paper tape punch, which was the most reliable data recording device when the design was completed in 1967. Magnavox has delivered 55 Geoceivers which are used primarily by the U.S. Defense Mapping Agency for geodetic survey work. The Geoceiver has earned an enviable reputation for accuracy and for reliability.

Figure 17 shows the latest Magnavox instrument intended for fixed point survey. It is called the MX 1502 Satellite Surveyor. Being

Figure 17. Magnavox MX 1502 Satellite Surveyor

compact and lightweight, it can be transported easily. In the field it will operate for about three days on a 12-volt automobile battery. During this time, the raw data from all satellite passes will be recorded on a magnetic tape cassette. The cassette can be processed by a computing center for either point positioning or translocation results.

The MX 1502 does far more than simply record satellite data. It computes and displays a 3-dimensional position fix result while in the field. This result often may be adequate without post-processing the tape cassette, but in any case it is extremely valuable in verifying proper system operation and assuring that the desired location has been occupied. In addition, the computed results help the surveyor to know when sufficient data have been gathered so that he can move to the next site with assurance. Assurance is a key ingredient of any survey system. Too often data are reduced to find that something was wrong and that the site must be reoccupied at great expense. The MX-1502 includes a thorough self-test capability to assure proper operation. If the self-test function detects a problem, the specific module causing the problem is indicated. Repair by replacement of plug-in modules allows the survey to continue with minimum disruption. Furthermore, after each record is placed on magnetic tape, it is immediately read back to assure no recording mistake. If an error is detected, that portion of data is re-recorded, always assuring that the proper data are recorded correctly.

The MX 1502 can learn the orbits of all Transit satellites by reading a previously recorded tape cassette. Thereafter, it will automatically go into a minimum power mode between satellite passes to reduce battery consumption, waking up just in time to track only the desirable passes. This new type of equipment will further expand the application of satellites, both for marine and for land surveys.

3.5.4 Point Positioning Accuracy

A single satellite pass can be used to obtain a latitude and longitude position fix result. As described in Chapter 6 of this document altitude must be defined, and an error in altitude can affect the posi-

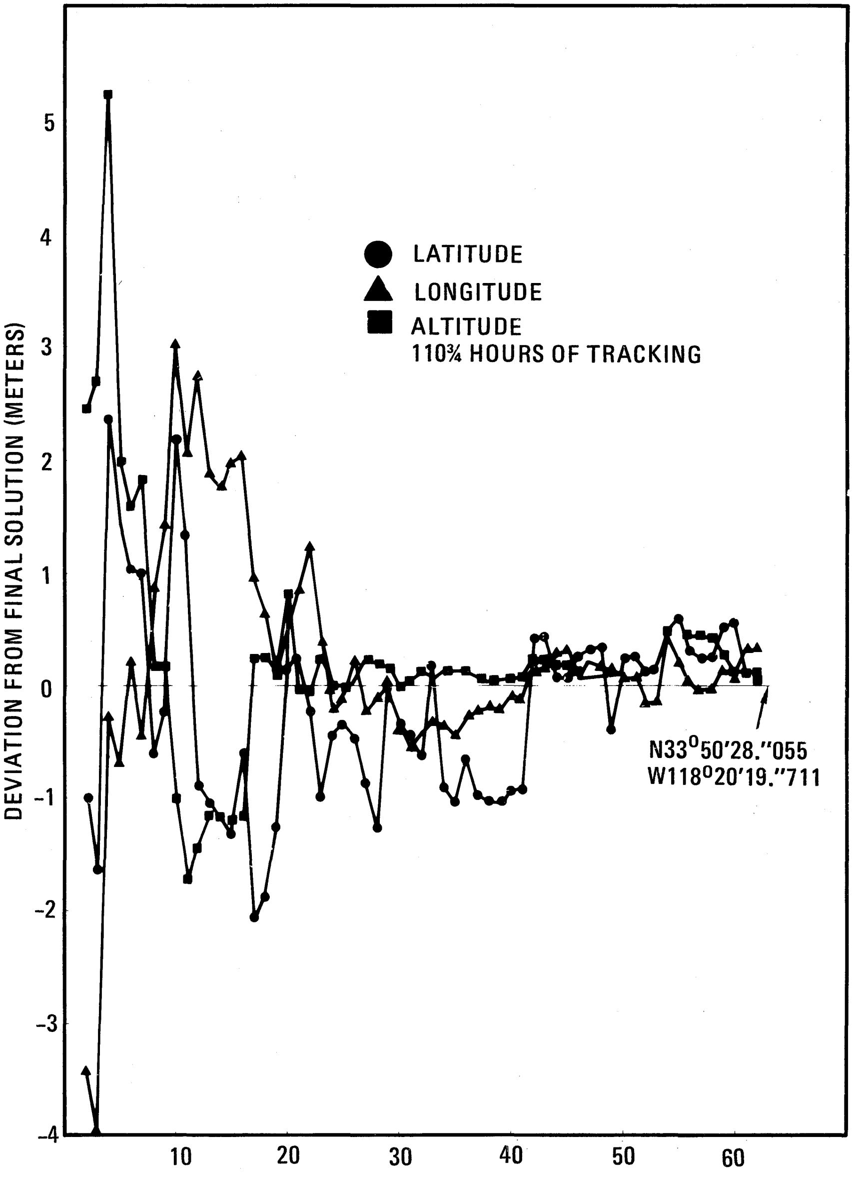


Figure 18. 3-D Point Positioning Convergence (62 MX 1502 Satellite Passes)

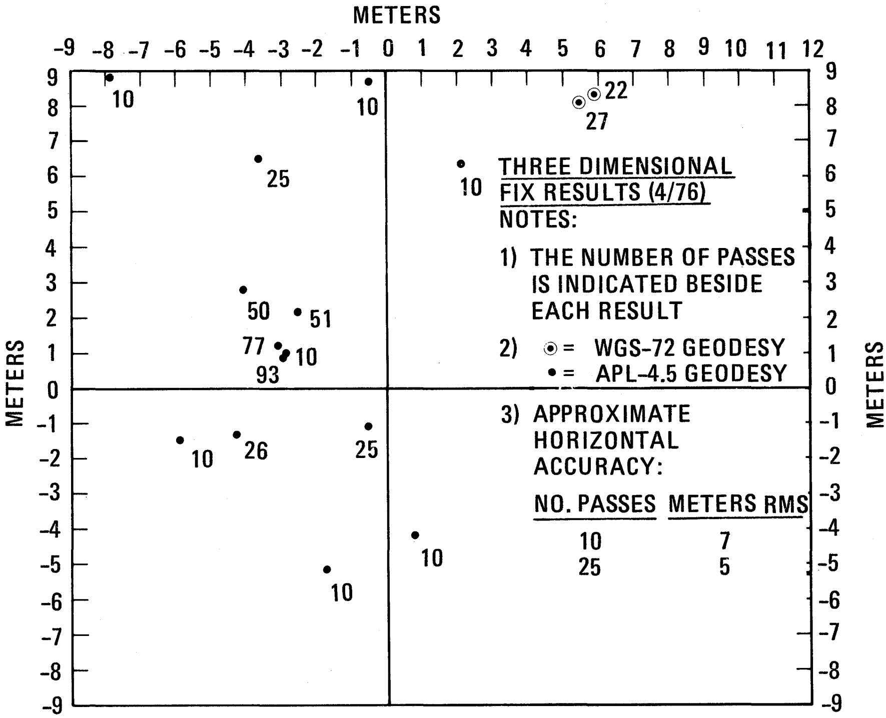


Figure 19. 3-D Point Positioning Results

tion fix accuracy. However, by processing multiple satellite passes at one location, a 3-dimensional (latitude, longitude, and altitude) position fix can be determined. The best way to do this is with a computer program which determines the one 3-dimensional position that best fits all of the Doppler measurements obtained from all of the satellite passes taken at that location. Figure 18 shows how a typical 3-dimensional survey converges toward the final answer. Each time another satellite is tracked, its data are combined with all previous data and a new solution is computed. The figure indicates that with each successive satellite pass, the latitude, longitude, and altitude parameters converge toward the final solution.

By conducting multiple convergence tests at the same location, one can determine the repeatability of the final solution. As expected, repeatability improves as the number of passes processed in each solution increases. Figure 19 makes this clear. The number beside each dot indicates how many passes were used for that position fix, and it is evident that, for example, there is more scatter to the 10-pass solutions than to the 25-pass solutions. As tabulated on the figure, the horizontal positioning repeatability is about 7 meters rms with 10 passes and about 5 meters rms with 25 passes.

Figure 19 also illustrates another important concept, which is that the position fix result is dependent on how the satellite orbits were determined. Most of the data shown in the figure was obtained before December 1975. In that month, the U.S. Navy changed the basis for computing satellite orbits from one model of the earth's gravity field to another (from APL-4.5 to WGS-72). The two points which are circled in the upper right of the figure were determined with the data taken after the conversion. Thus, we can use the term "accuracy" only if we accept the satellite system as the basic geodetic reference. Otherwise, it is proper only to describe the repeatability of such a process.

The results just described are available to every system user with the necessary equipment and computer program. The principal source of error is misknowledge of satellite orbital position, made worse by the fact that orbit parameters in the satellite memory are a prediction of its position based on past tracking data. The prediction is obtained by numerical integration of the equations of motion, taking into account all known forces acting on the satellite, such as the gravity fields of the earth, sun, and moon, plus drag and radiation pressures. To the extent that these forces are not known precisely, the predicted orbit will deviate from the actual orbit. These differences account for most of the 27 to 37 meters rms of error in individual Transit position fixes.

If the orbit did not have to be predicted into the future, a more precise determination could be made, and the U.S. Defense Mapping Agency (DMA) employs this technique in reducing satellite Doppler data from survey receivers such as the AN/PRR-14 Geoceiver. Field data are recorded on tape and returned to a computing center for evaluation. There the Doppler data are combined with a precise ephemeris of satellite positions based on tracked rather than predicted orbits; thus individual position fixes have a typical scatter of only 6.3 meters rms. Naturally a 3-dimensional, multi-pass solution converges to the required resolution much faster with this technique than when using predicted orbit parameters from the satellite. However, the DMA seldom computes a precise ephemeris for more than one or two satellites at a time, and immediate results cannot be obtained in the field, offsetting slightly the advantage just described. Even so, equipment using the predicted orbits must remain on station from 4 to 10 times longer than equipment using the precise ephemeris for equivalent accuracy results. The DMA has shown 3-dimensional results with 1.5 meters per axis repeatability after 25 precise ephemeris passes.

Precise ephemeris information is not available for commercial use. However, there is precedent for the DMA to supply this information to other nations based on cooperative international survey agreements.

Unfortunately, there is evidence that a precise ephemeris position fix result will differ from one using data from the satellite message. This difference is because the DMA uses a slightly different gravity model to compute satellite orbits than does the Navy Astronautics Group. This author regrets the difference and does not understand why it must persist.

3.5.5 Translocation Accuracy

Although precise ephemeris data are not available commercially, another technique called translocation can yield equivalent results. Advantage is taken of the fact that almost all the error in a position fix is caused by factors external to the satellite receiver. Thus, two receivers tracking the same satellite pass at the same location should produce nearly the same result (i.e., the errors are strongly correlated). Experience has shown that the correlation is quite effective for interstation separations of 200 km or more. As a result, two or more stations can be located with respect to each other with an accuracy of 1 meter or better over very considerable distances.

One method of using translocation is to establish a base station which collects data from all available satellite passes for days or

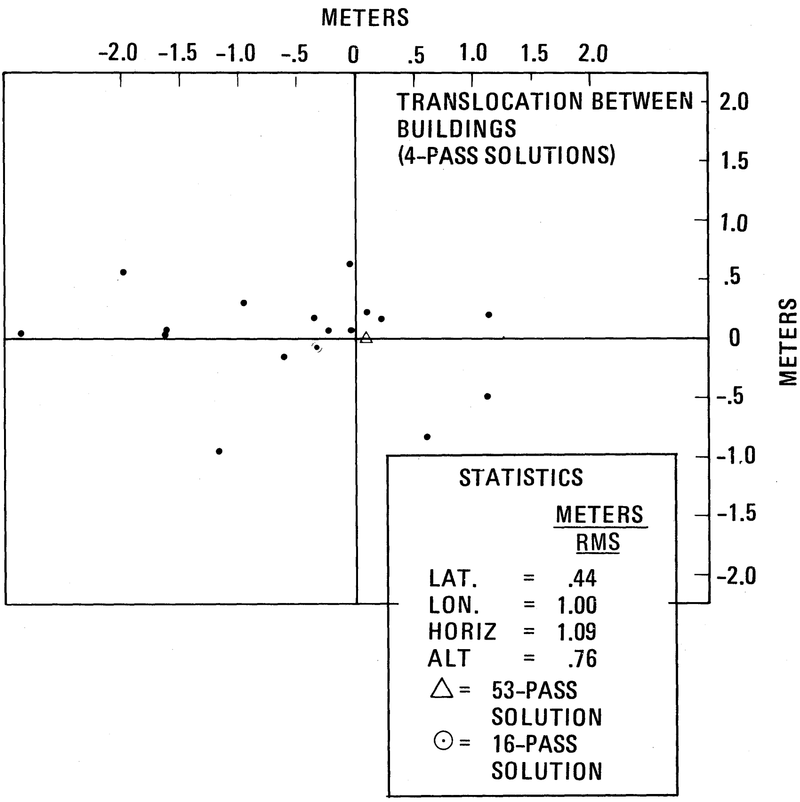


Figure 20. 4–Pass 3–D Translocation Results

weeks. When fed to the 3-dimensional point positioning program, these data will yield an excellent absolute position determination. In the meantime, one or more portable receivers move from one location to another gathering 8 to 10 passes at each site. These data are then processed by translocation to define the position of each remote site with respect to the accurate base station location. An equally valid concept is to locate one station on an established and accepted geodetic reference point, thus using translocation to carry this geodetic reference to the remote sites.

Figures 20 and 21 show translocation results between two antennas which were very near each other so their relative position could be

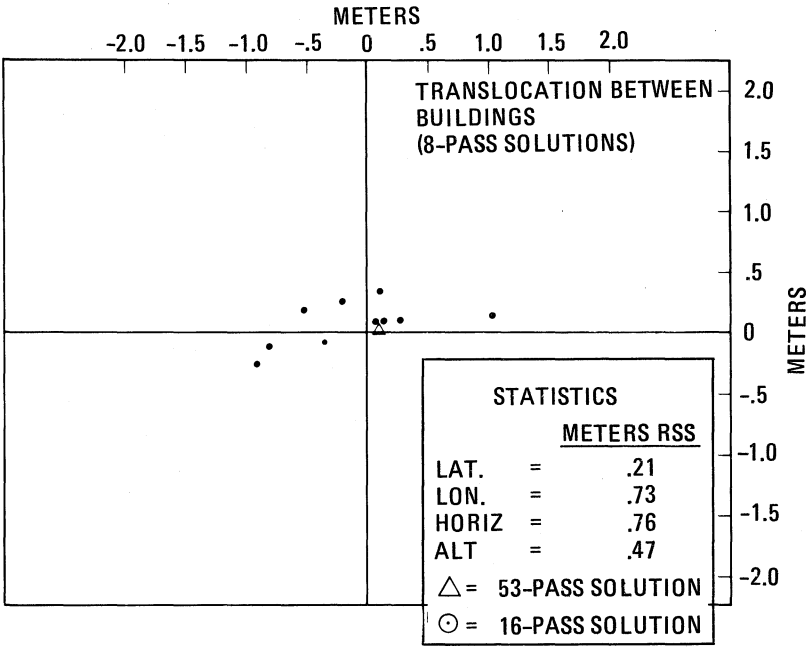


Figure 21. 8-Pass 3-D Translocation Results

determined with great accuracy. Each dot shows the difference between the translocation result and the survey reference. All satellite passes above 15 degrees maximum elevation were used. For this test, manual editing forced a balance of east and west passes for the 4-pass solutions. For the 8-pass solutions, an imbalance of 5 vs 3 was allowed. Otherwise, all other editing was performed automatically. The horizontal accuracy was 1.09 meters rms for the 4-pass solutions and 76 centimeters rms for the 8-pass solutions. This is a measure of quality both of the computer program and of the receivers being used for the test. It should be noted that slightly better results could be obtained through use of a rubidium or cesium frequency standard at each receiver. Field tests indicate that this level of translocation accuracy is obtainable over distances of several hundred kilometers.

Figure 22. AN/WRN-5 Military Satellite Navigator

3.6 MILITARY APPLICATIONS

The Transit system was developed initially to provide precise position updates for the Polaris submarine fleet. In this application, a submarine will expose its antenna at the appropriate time to update and to maintain the accuracy of its inertial navigation systems. Transit continues to be operated specifically to serve this Navy application .

U.S. Navy attack submarines also are navigated by Transit. Figure 22 shows the AN/WRN-5 satellite navigator which was developed for use aboard nuclear attack submarines, although more are now being used aboard surface ships. A number of other Transit sets also are being used to navigate attack submarines, including the MX 702A/HP system shown in Figure 12 and, more recently, the MX 1102 Satellite Navigator shown in Figure 11. In fact, several NATO navies have expressed interest in a combination Transit-Omega navigator implemented within the MX 1102 structure both for submarines and for surface ships.

Submarine applications require the Transit navigator to provide satellite alert information so that appropriate times can be chosen to expose the antenna. In addition, it is desirable to minimize the duration of each antenna exposure. This requires a receiver such as the MX 1102 which tunes to the proper satellite frequency automatically, otherwise some provision for manual tuning must be provided.

Rather than tracking only selected satellite passes, surface ships track every available satellite pass. The navigation concepts, applications, and advantages are the same as for commercial ships, except that accurate, worldwide, all weather navigation also provides tactical and strategic advantages. Applications range from the navigation of major combat ships to patrol vessels guarding the 200 mile economic zone boundary.

Transit is used extensively for military land survey and mapping purposes. The U.S. Defense Mapping Agency and many of the NATO nations have cooperated on satellite survey operations across Europe. Equipment such as the AN/PRR-14 Geoceiver, shown in Figure 16, and the MX 1502 Satellite Surveyor, shown in Figure 17, can be used for these purposes.

As Transit user equipment has become smaller, more reliable, and less expensive, the opportunity for other land applications has been created. Magnavox is investigating the application of Transit fixes to vehicle positioning and even to manpack use. Although the time interval between Transit fixes is not desirable, there are many situations in which Transit could well be the only source of accurate geographic reference. This is particularly true for vast desert or jungle areas where accurately surveyed landmarks are not readily available.

CHAPTER 4
TRANSIT STATUS AND VITALITY

4.1 HISTORY AND FUTURE

Development of Transit began late in 1958, and the system became operational in January of 1964. On July 29, 1967, then Vice President Hubert H. Humphrey made an important announcement as part of a speech at Bowdoin College. The key paragraph from this speech reads as follows:

> "This week the President approved a recommendation that the Navy's Navigation Satellite System be made available for use by our civilian ships and that commercial manufacture of the required shipboard receivers be encouraged. This recommendation was developed by the Department of the Navy in support of initiatives of the Marine Sciences Council to strengthen worldwide navigational aids for civilian use. Our all-weather satellite system has been in use since 1964 by the Navy and has enabled fleet units to pinpoint their positions anywhere on the earth. The same degree of navigational accuracy will now be available to our non-military ships."

The use of Transit has expanded greatly in the years since its introduction. Manufacturers around the world have taken the Presidential encouragement literally, and since 1968 when the first commercial Transit sets were available, there has been a steady and dramatic increase in the types of equipment available and the number of users worldwide.

Regardless of past achievements, however, questions are raised about the future of Transit now that NAVSTAR, the Global Positioning System (GPS) is being developed. If GPS achieves its development objectives and operational funding is approved by the U.S. Congress, it is reasonable to expect that Transit will be discontinued after a sufficient overlap interval for users to depreciate existing equipment and to select appropriate replacement GPS equipment. Although no policy statement has been published at this time, the available information (see Reference 12) makes this author conclude that

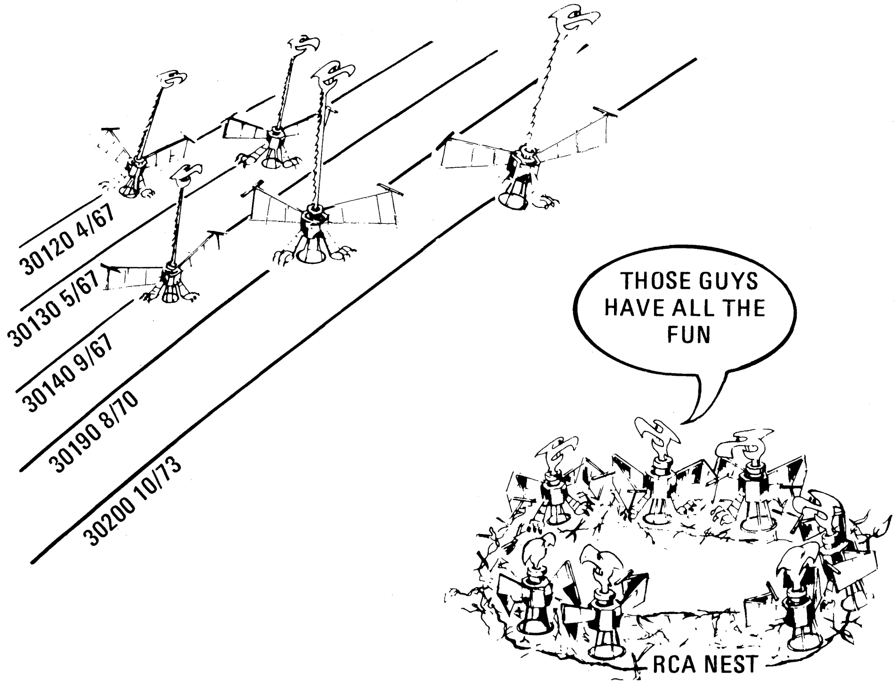


Figure 23. The 5 Operational Transit Satellites, Launched on the Dates Shown, are Backed by Twelve Reserve Spacecraft at RCA

Transit will be available until at least 1995. The following paragraphs emphasize the vitality of the Transit system today and for the foreseeable future.

4.2 SYSTEM RELIABILITY AND AVAILABILITY

The Transit system reliability and availability can be seen in a number of areas. One is the remarkable success rate of the Navy Astronautics Group in maintaining a proper orbit message in the memory of each satellite. From January of 1964 to April of 1977, there had been only 7 message injections which were not verified as 100 percent successful out of a total of 32,389 attempts. Each of the 7 was corrected on the next satellite pass, about 107 minutes later. This is a 99.98 percent success record and shows outstanding system reliability.

Figure 23 expresses the satellite status in terms of reliability and availability. Three of the five operational satellites were launched over ten years ago at this writing. Amazingly, the signals are strong and the satellites continue to function flawlessly. Backing up this group of "never say die" performers are twelve spacecraft stored where they were built many years ago at RCA Astro Electronics in New Jersey.

Being very light (about 61 kilograms), Transit satellites can be placed in their 1,100 kilometer orbits with relatively inexpensive, solid fuel Scout rockets. Nine of these boosters currently are in reserve to support future launches.

It appears that Transit is in extremely good health when it comes to reliable performance today and provision for continuation of service for many years to come, especially noting the proven longevity of the spacecraft design.

4.3 NEW GENERATION OF SATELLITES

As shown in Figure 24, the Applied Physics Laboratory has developed a new generation of Transit satellites, which they called TIP for Transit Improvement Program. Two prototype satellites were launched as part of the development effort.

The Navy has decided to produce a limited number of these new satellites, which will now be called NOVA. RCA is building the first three NOVA satellites, and it is expected that at least two more will be built. The first NOVA is expected to be launched in the third quarter of 1979. This new satellite will be especially welcome in filling the orbit gap now existing between satellites 30120 and 30200, as discussed in Section 4.7.

The NOVA satellite signals are entirely compatible with the existing Transit satellite signals. Therefore, all users will have access to this new spacecraft. However, the NOVA satellites provide many important new capabilities, all of which have been verified with the experimental TIP satellites. Of particular interest are the following:

- DISCOS, for disturbance compensation system, eliminates

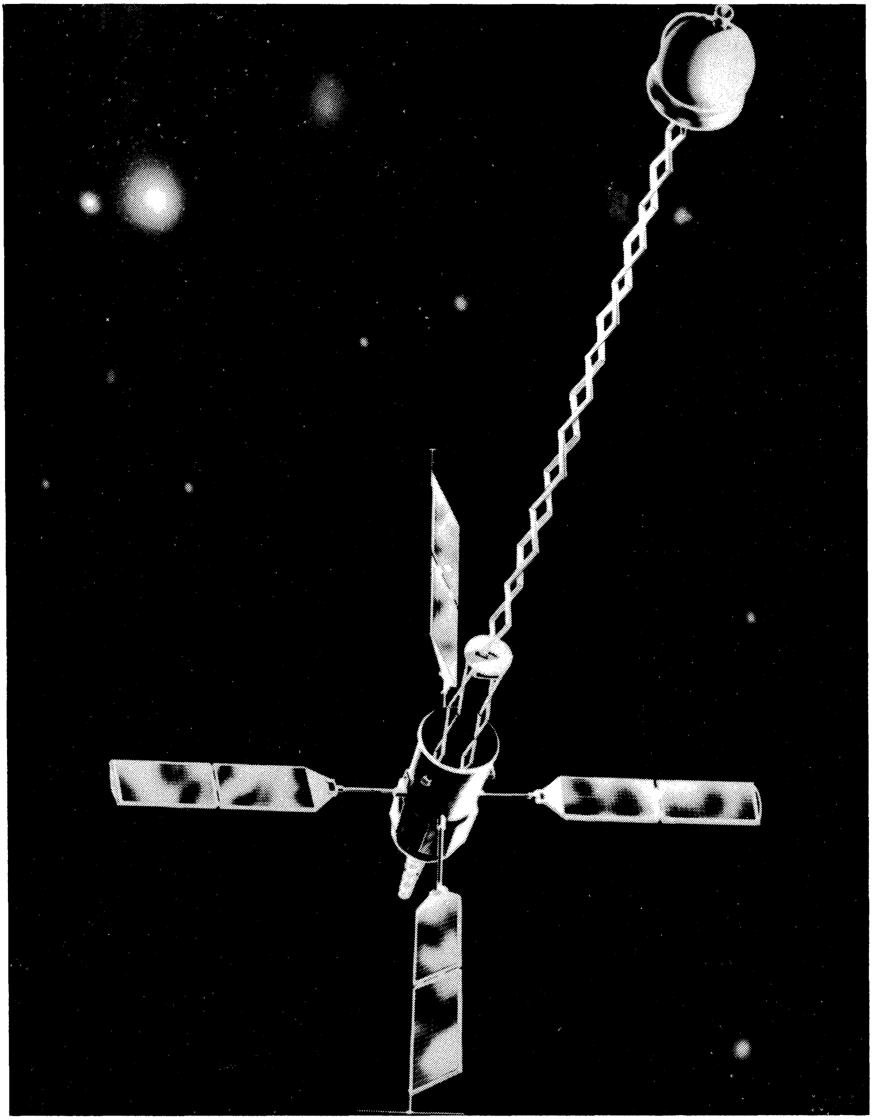


Figure 24. New Generation NOVA Transit Satellite (Previously called TIPS)

the effect of atmospheric drag. As a result, each orbit determination will retain accuracy for up to a week instead of 24 hours now. With NOVA, we expect survey navigation results to converge faster and have better accuracy.

- NOVA is controlled by an on-board general purpose digital computer which can be programmed from the ground. In conjunction with a larger memory, the computer can provide orbit parameters for ten days without requiring upload of new information.

- A new data modulation, transparent to existing receivers, can be switched on. Plans for this modulation have not been announced, but it could be used to provide more precise orbit parameters.

- The received signal level from NOVA satellites will be twice as strong (3 dB). Antenna polarization will be left hand circular on both channels rather than left on 150 MHz and right on 400 MHz at present.

- Very precise clock control has been achieved by permitting the onboard computer to adjust oscillator frequency with a resolution of about 1×10^{-12} (To make the carrier and the data modulation coherent, the nominal frequency offset has been changed from 80 ppm to 84.48 ppm, which should not cause compatibility problems.)

- To transmit the precise time information, a high frequency pseudo-random noise (PRN) modulation has been added to both the 150 and 400 MHz signals. This also can be used to achieve single-channel, refraction corrected fixes (by detecting the difference in group delay and phase delay effects), and a properly equipped receiver can block out signals from any other satellite, thus eliminating the potential for cross-satellite interference.

4.4 EXPANDING USER BASE

Figure 25 is a chart prepared by the Navy Astronautics Group based on information received from 15 of 19 manufacturers of Transit user equipment. The chart shows a total user population of 1,899 sets at the beginning of 1977, which was expected to grow to 4,350 sets by the end of 1978.

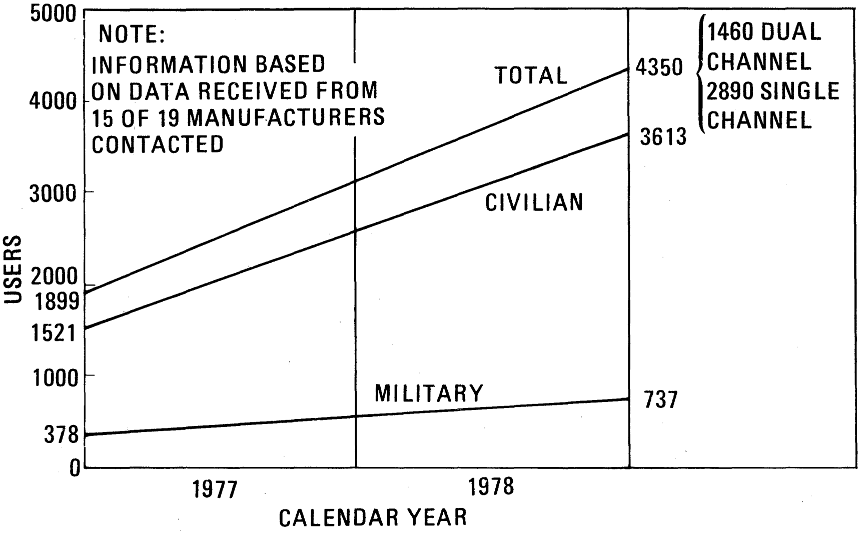


Figure 25. Present Status and Expected Growth in Number of Transit System Users (Provided by the Navy Astronautics Group)

The user population growth predicted by the manufacturers represents an annual growth rate of 51 percent. To see if this were possible, data was included from an earlier survey showing the total population at the beginning of 1974 to be 600 sets. Growing from 600 to 1,899 in three years required an annual rate of 47 percent. Thus, the predicted annual growth of 51 percent appears to be in line with past trends, and it may be conservative when recent product innovations are considered.

Figure 25 shows the growth as a linear function of time, but including the data from 1974 tells us that this is not the case. In fact, the number of users has been increasing as a percentage of the existing population, which is a straight line on logarithmic paper. Figure 26 is such a plot using the three data points provided by the Navy Astronautics Group. What may be surprising is that at present rates the user population should reach 10,000 by the early 1980's. Based on data available as of the first quarter of 1978, this growth trend appears to be continuing.

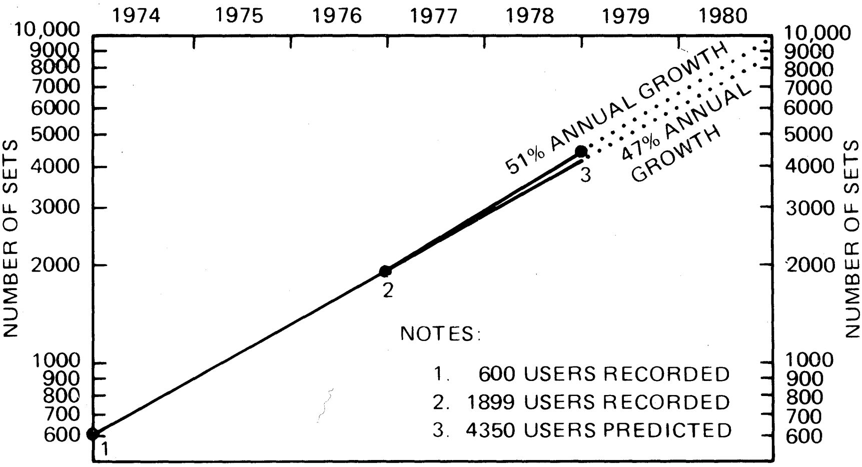


Figure 26. Growth of Transit User Population Obtained From Data Provided by the Navy Astronautics Group

4.5 INVESTMENT IN TRANSIT NAVIGATION EQUIPMENT

Combining data from the Navy Astronautics Group with other sources, the total investment in Transit navigation equipment has been estimated, as summarized by Figure 27. Research and development costs are not included, and equipment known to be out of service has been deleted. Overall, we believe the estimates are on the low side.

The Navy Strategic Systems Project Office has been included as a separate category due to their special involvement with Transit. The total U.S. Government investment in Transit user equipment is nearly 45 million dollars. Most of the integrated systems are owned and operated by private firms engaged in offshore oil exploration. The remaining dual-channel navigation systems are used for survey work of various types, such as oceanography, land survey, drill rig positioning, cable laying, etc. The single-channel navigators are used for general navigation purposes where 0.1 mile (fix accuracy is sufficient, and this is the area of fastest growth.

CATEGORY	QUANTITY	AV. COST (THOUSANDS)	TOTAL COST (MILLIONS)	WITH SPARES (MILLIONS)
NAVY STRATEGIC SYSTEMS PROJECT OFFICE	73	$ 251	$ 18.4	$ 23.9
U.S. GOVERNMENT - ALL OTHER	469	56	26.3	34.2
INTEGRATED SYSTEMS	118	231	27.3	35.5
OTHER DUAL-CHANNEL	539	47	25.2	32.8
SINGLE-CHANNEL	2239	22	48.4	53.2
TOTALS	3438		$145.6	$179.6

Figure 27. Estimated Investment in Transit Navigation Equipment (April 1978)

The last column in Figure 27 is an estimate of the cost of equipment plus spares. Ten percent spares cost was assumed for the single-channel equipment and 30 percent for all other categories.

Figures 26 and 27 carry a powerful and surprising message. It is probable that at this time more money has been invested in Transit user equipment than in marine equipment for any other U.S. radionavigation system, including Loran-A, Loran-C, or Omega. Naturally the reason for this has been the much higher price for Transit equipment, which always requires a computer and often is combined with other sensors to form an integrated system. However, Figure 26 shows that the user population also is growing rapidly, spurred by technical innovations which permit lower prices, better performance, and greater reliability.

4.6 COST OF TRANSIT SYSTEM OPERATION

The cost of operating Transit has been estimated by the Navy to be as shown in Figure 28. For those familiar with the operational costs of any other major navigation system, it should be obvious that Transit is very inexpensive to operate and to maintain.

4.7 IMPROVEMENT IN ORBITAL COVERAGE

Figure 29 shows the orbital spacing of the five operational Transit satellites and their rates of precession as of March 23, 1978. This specific orbital configuration was used to predict the average interval between satellite fixes given by Figure 3.

TRANSIT GROUND STATION	PERSONNEL
POINT MUGU, CALIFORNIA	152
PROSPECT HARBOR, MAINE	20
ROSEMONT, MINNESOTA	28
WAHIAWA, HAWAII	9
TOTAL	209

ANNUAL SUPPORT	ANNUAL COST
TRANSIT GROUP SUPPORT	$ 5.0 M
STORAGE OF 12 SATELLITES	0.3

SATELLITE REPLACEMENT COST	EACH
(INCLUDES SCOUT LAUNCH VEHICLE, SATELLITE CHECKOUT AND LAUNCH SUPPORT.)	$ 3.5 M

Figure 28. Cost of Operating the Transit System (Provided by the U.S. Navy, April 1977)

A better way to visualize the interval between fixes is that of Figure 30, which shows the cumulative waiting time probability at three different latitudes. Note that intervals of more than 12 hours occur infrequently at the equator, and intervals of six to seven hours occur at higher latitudes. These peak values are strongly related to the large gap between satellites 30120 and 30200 shown in Figure 29, which is growing at about 5.1 degrees per year.

To evaluate the effect of filling the gap with another satellite, the interval prediction program also was run with six satellites. The sixth satellite is TRANSAT (30110), shown with a dotted line in Figure 29, which was launched by the U.S. Navy in 1977. This satellite is intended for purposes other than navigation, although it has a Transit navigation mode which can be switched on if desired.

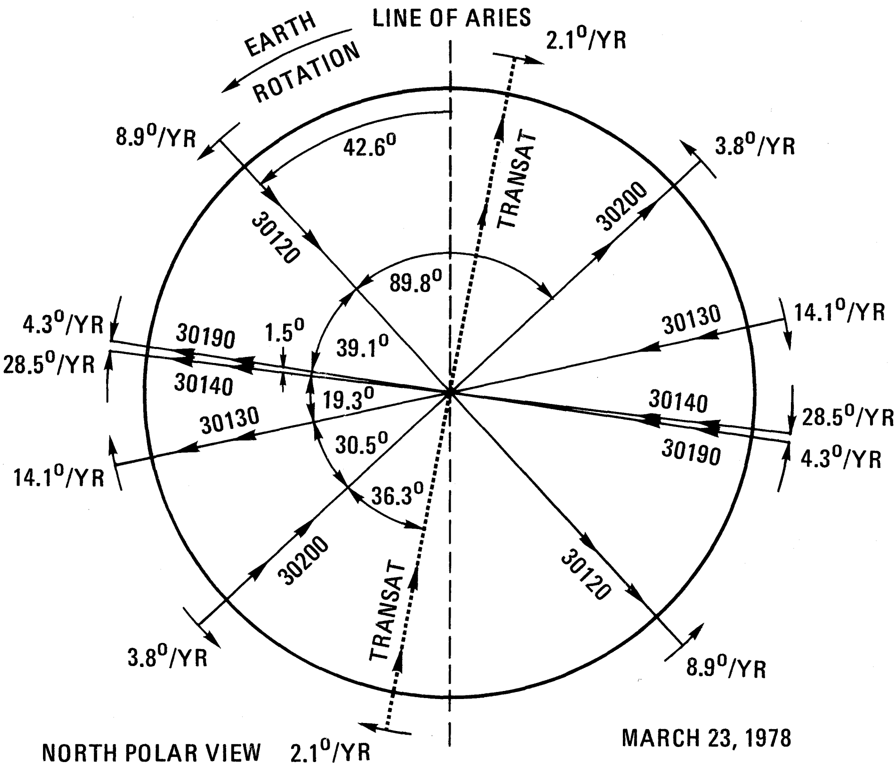


Figure 29. Orbital Separation of the Five Operational Transit Satellites and TRANSAT (30110) on March 23, 1978

Figure 31, when compared with Figure 30, shows the dramatic effect of having a satellite in the orbit coverage gap. Not only are there more satellite fixes available, but a much higher percentage occur after shorter waiting times. Figure 32 shows the effect on mean time between fixes of having TRANSAT.

Although having the gap filled would be very desirable, the Navy does not plan to use TRANSAT in this way. However, as described in Section 4.3, the Navy does plan to launch the new generation of NOVA satellites beginning in the third quarter of 1979. Not only

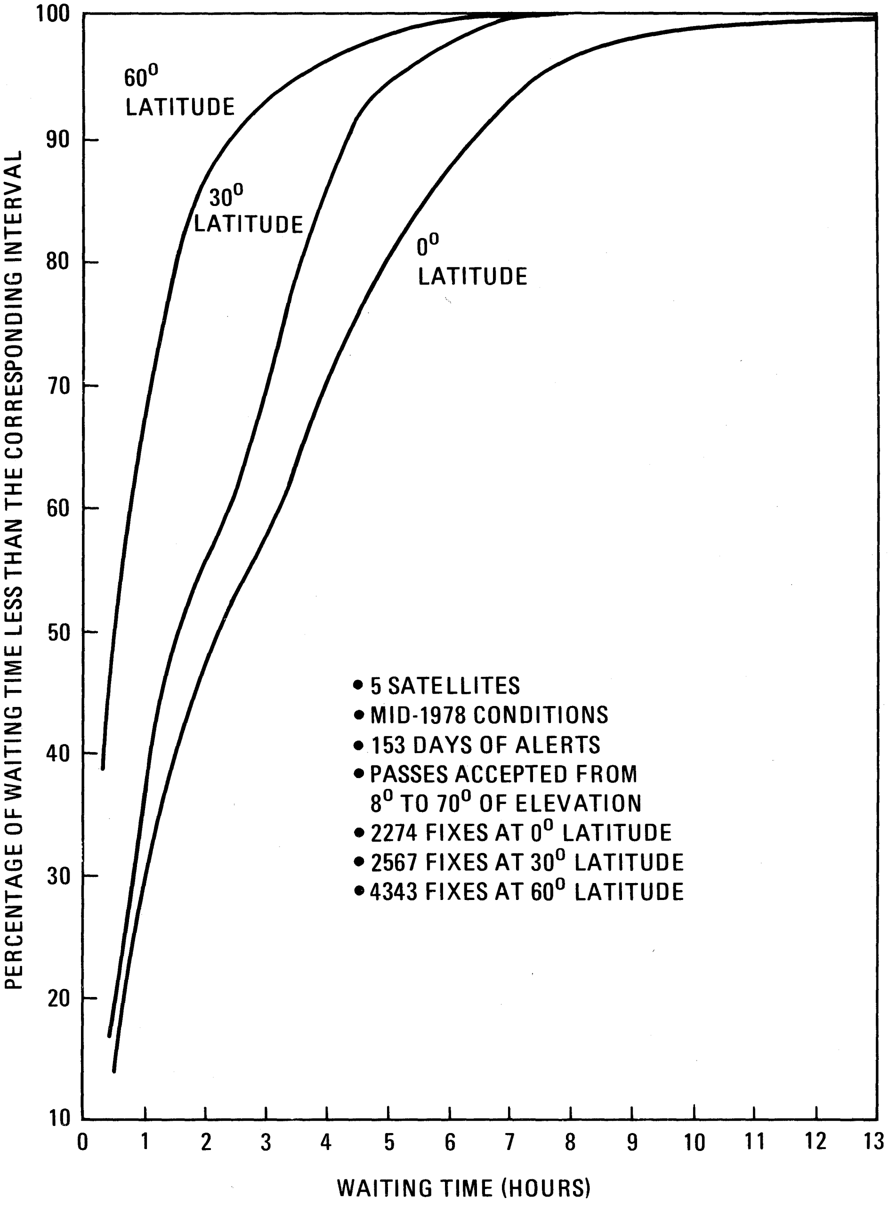


Figure 30. Cumulative Probability of Waiting Time for the Next Transit Fix With the Five Current Satellites (mid-1978)

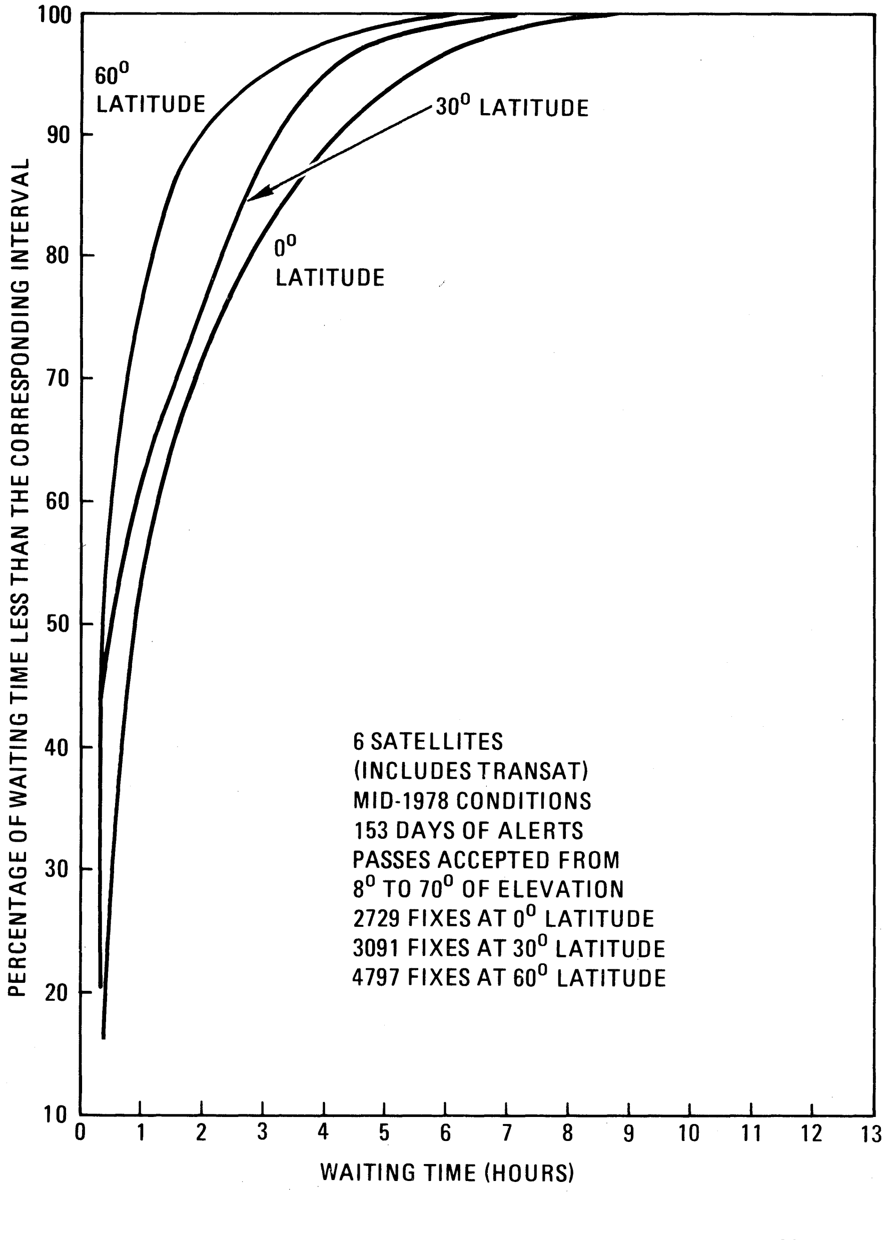


Figure 31. Cumulative Probability of Waiting Time for the Next Transit Fix Assuming TRANSAT Use (mid-1978)

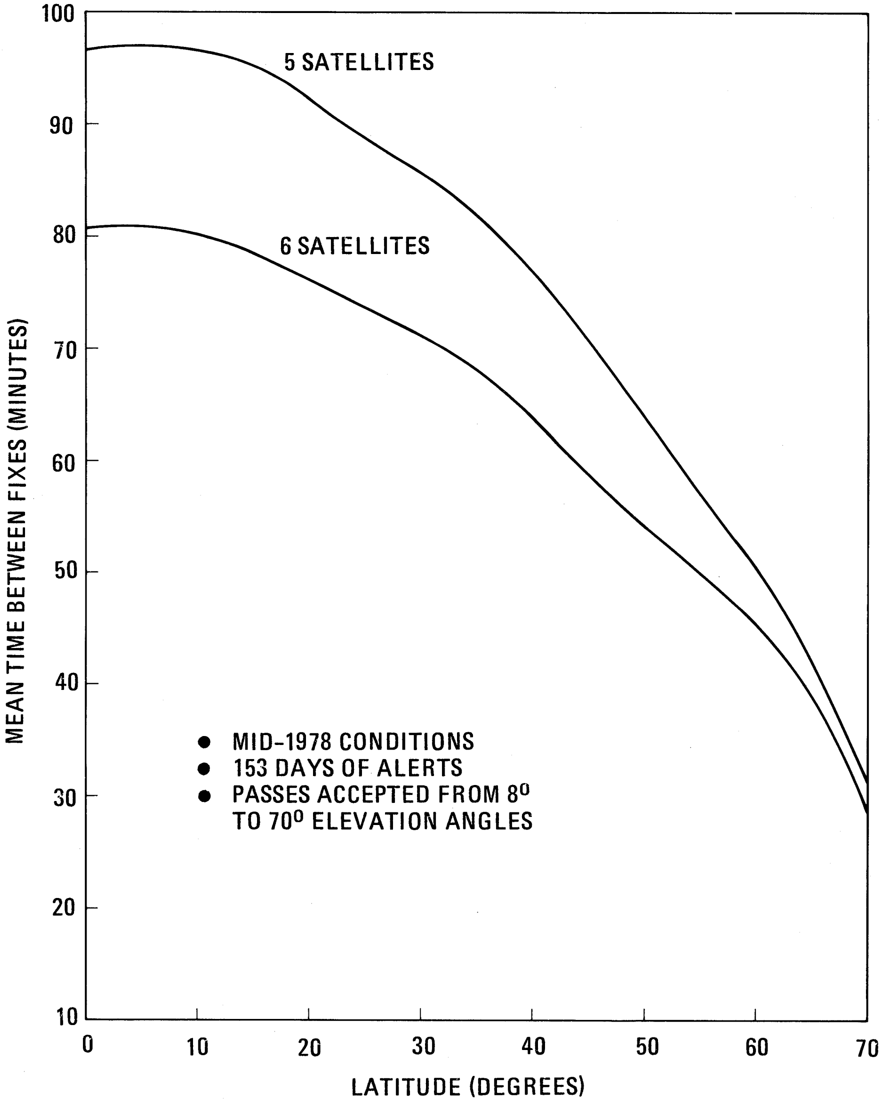


Figure 32. Mean Time Between Fixes Which Would Occur With and Without TRANSAT During mid-1978

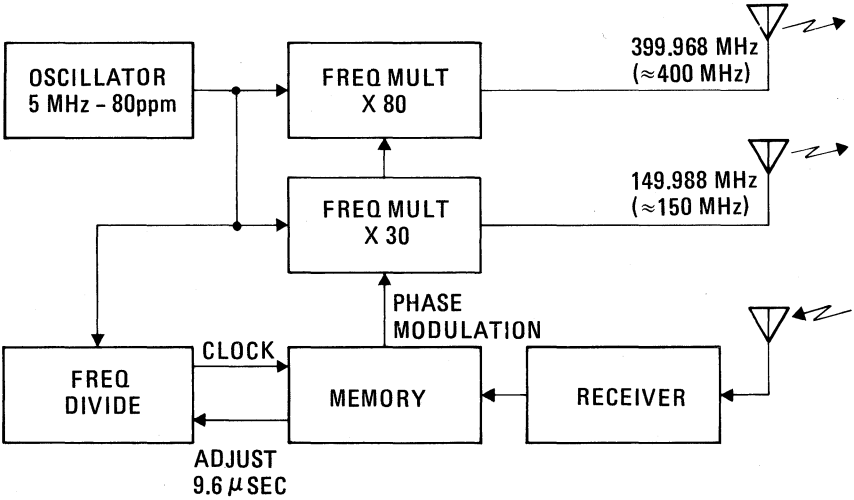


Figure 33. Transit Satellite Block Diagram

will NOVA fill the gap, but the orbits will be controlled to maintain precession at negligible levels. In 1980, two NOVA satellites with orthogonal orbits will form the backbone of the Transit system, with the existing satellites continuing to provide fixes as well.

4.8 SUMMARY

The preceding paragraphs have attempted to communicate the basic vitality of the Transit system. We see this vitality in the system reliability, the new generation of satellites, the expanding user base, the amazing breadth of applications, the substantial worldwide investment in Transit navigation equipment, and in the very low cost of system operation. With all things considered, this author is certain the Transit system will continue to provide its vital navigation service until at least 1995.

CHAPTER 5
THE POSITION FIX TECHNIQUE

5.1 THE SATELLITE SIGNALS

Figure 33 is a block diagram of the Transit satellite electronics. The satellites transmit coherent carrier frequencies at approximately 150 and 400 MHz. Because both signals are derived by direct multiplication of the reference oscillator output, the transmitted frequencies are very stable, changing no more than about 1 part in 10^{11} during a satellite pass. Thus, they may be assumed to be constant with negligible error.

The reference oscillator output also is divided in frequency to drive the memory system. In this way, the navigation message stored there is read out and encoded by phase modulation onto both the 150 and 400 MHz signals at a constant and carefully controlled rate. Thus, the transmitted signals provide not only a constant reference frequency and a navigation message but also timing signals, because the navigation message is controlled to begin and to end at the instant of every even minute. An updated navigation message and time corrections are obtained periodically from the ground by way of the satellite's injection receiver. The time correction data are stored in the memory and applied in steps of 9.6 microseconds each.

Each binary bit of the message is transmitted by phase modulation of the 150 and 400 MHz signals. The modulation format for a binary one is given in Figure 34, and a binary zero is transmitted with the inverse pattern. As shown, this format furnishes a clock signal at twice the bit rate, which is used to synchronize the receiving equipment with the message data.

Because the satellites transmit only about one watt of power and may be thousands of kilometers away, very sensitive receivers are needed. In addition, however, the orbit parameters must be verified by comparing redundant messages to detect and eliminate occasional errors in the received data.

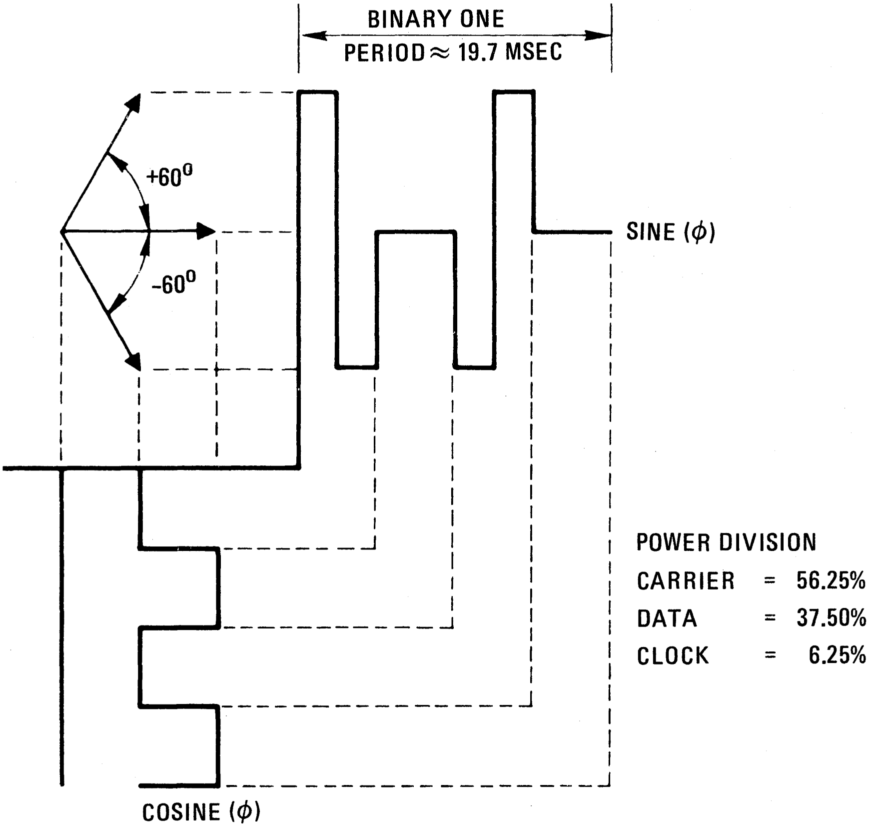
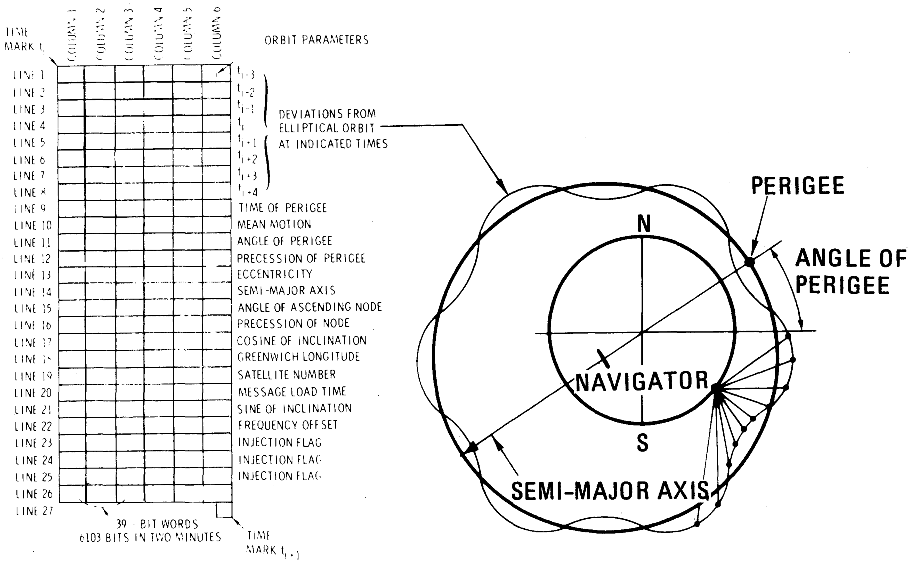


Figure 34. Transit Data Phase Modulation

5.2 INTERPRETATION OF SATELLITE MESSAGE

Figure 35 indicates how the navigation message defines the position of the satellite. During every two minute interval the satellite transmits a message consisting of 6,103 binary bits of data organized into 6 columns and 26 lines of 39-bit words, plus a final 19 bits. The message begins and ends at the instant of the even minute, which are denoted as time marks t_i and t_{i+1}. The final 25 bits of each message form a synchronization word (0111111111111111111111110) that identifies the time mark and the start of the next 2-minute message. By recognizing this word, the navigation receiver establishes time synchronization and thereafter can identify specific message words.



TWO MINUTE MESSAGE ORGANIZATION **ORBIT DEFINITION**

Figure 35. Satellite Message Describes Orbital Position

The orbital parameters are located in the first 22 words of column 6, and those in lines 9 through 22 are changed only when a new message is injected into the memory. These fixed parameters define a smooth, precessing, elliptical orbit; satellite position being a function of time since a recent time of orbit perigee.

The words in lines one through eight shift upward one place every two minutes, with a new word inserted each time in line eight. These variable parameters describe the deviation from the smooth ellipse of the actual satellite position at the indicated even minute time marks. By interpolation through the individual variable parameters, the satellite position can be defined at any time during the satellite pass.

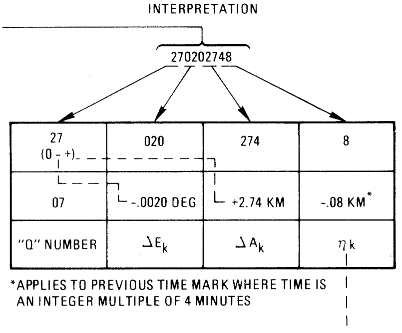
Figure 36 aids in interpreting the Transit message parameters. On the left is a set of typical fixed parameters and an indication of how they are to be interpreted. On the right is a set of variable parameters with an interpretation of one. The following paragraphs will describe how each of these is used.

TYPICAL
SATELLITE MESSAGE
FIXED PARAMETERS

049160940	TIME OF PERIGEE = 491.6094 MINUTES
836540260	MEAN MOTION = 3.3654026 DEG/MIN
815801870	ARGUMENT OF PERIGEE = 158.0187 DEG
800198330	RATE OF CHANGE OF ABOVE = .0019833 DEG/MIN
800022690	ECCENTRICITY = 0.002269
807464570	SEMI-MAJOR AXIS 7464.57 KM
803673600	RIGHT ASCENSION OF ASCENDING NODE = 36.7360 DEG
900002840	RATE OF CHANGE OF ABOVE = -.0000284 DEG/MIN
800067000	COSINE OF INCLINATION = 0.006700
814855960	RIGHT ASCENSION OF GREENWICH = 148.5596 DEG
809999780	SINE OF INCLINATION = 0.999978

TYPICAL SATELLITE
VARIABLE PARAMETERS

250512804
260362810
270272748
280062604
090072400
400182134
410261833
420321504
430341164
440330834
000290534
010220284
020130084
130020044



*APPLIES TO PREVIOUS TIME MARK WHERE TIME IS AN INTEGER MULTIPLE OF 4 MINUTES

BCDXS3 CODE

0011	=	0	1000	=	5
0100	=	1	1001	=	6
0101	=	2	1010	=	7
0110	=	3	1011	=	8
0111	=	4	1100	=	9

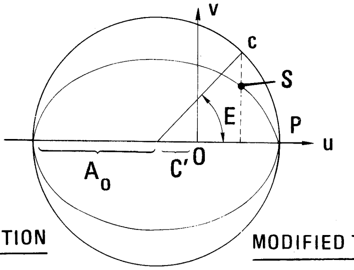
MEANING OF FIRST DIGIT

0	=	++0	5	=	+-1
1	=	+-0	6	=	-+1
2	=	-+0	7	=	--1
3	=	--0	8	=	+
4		++1	9	=	-

FIRST DIGIT OF ηk

0	=	-0	5	=	+0
1	=	-4	6	=	+1
2	=	-3	7	=	+2
3	=	-2	8	=	+3
4	=	-1	9	=	+4

Figure 36. Interpretation of the Transit Message Parameters



$$\epsilon = \frac{C'}{A_0}$$

CLASSICAL ORBIT DEFINITION

$M(t) = n(t-t_p)$

$E(t) = M(t) + \epsilon \text{ SIN } E(t)$

$A = A_0$

$u(t) = A(\text{COS } E(t) - \epsilon)$

$v(t) = A\sqrt{1-\epsilon^2}\text{SIN } E(t)$

w(t) IS UNDEFINED

O = EARTH CENTER
S = SATELLITE POSITION
P = PERIGEE
M = MEAN ANOMALY
E = ECCENTRIC ANOMALY
A_0 = SEMI-MAJOR AXIS

MODIFIED TRANSIT ORBIT DEFINITION

$M_k = n(t_k - t_p)$

$E_k = M_k + \epsilon \text{ SIN } M_k + \Delta E(t_k)$

$A_k = A_0 + \Delta A(t_k)$

$u_k = A_k(\text{COS}(E_k) - \epsilon)$

$v_k = A_K \text{ SIN } (E_k)$

$w_k = \eta(t_k)$

Figure 37. u, v, w Satellite Coordinates are Earth-Centered and Aligned with Perigee

The "Q" number provides a time tag for each word of the variable parameters. In the example given, the number 07 means that this word applies at seven 2-minute intervals past the half hour, i.e., 14 minutes or 44 minutes after the hour. This is why it is necessary to initialize a Transit set to within plus or minus 15 minutes of correct (GMT) time in order to synchronize properly. A time error of less than 15 minutes will be resolved by the "Q" numbers from the satellite message.

From Figure 36 also note that only one digit of the variable parameter η_k is transmitted in each word. Because two digits are required, this parameter is defined only every four minutes at times which are integer multiples of four minutes. The interpretation of the first digit of η_k also is given by the figure.

The objective is to define the satellite position as a function of time. To achieve this, three different coordinate systems are employed. Figure 37 defines the u, v, w coordinate system. These coordinates are earth-centered, u and v lie in the plane of the satellite orbit, and u is through the point of perigee (closest point to the earth). On the left of Figure 37 are shown the classical Kepler orbit definition equations. The Transit orbit definition equations are very similar, except for simplifications in the expressions for E_k and for v_k. Error introduced by these simplifications is eliminated by application of variable parameters $\triangle E_k$ and $\triangle A_k$. The w_k parameter defines out-of-plane satellite motion, which is simply the variable parameter η_k.

Figure 38 shows how the x′, y′, z′ coordinates are obtained by rotation of the u, v, w coordinates. Rotation by the "argument of perigee" places x′ in the earth's equatorial plane.

Finally, Figure 39 shows that with two rotations the satellite position can be defined in an X, Y, Z coordinate system which is earth-centered and earth fixed, with Z being the polar axis (mean pole of 1900-1905 or Conventional International Origin) and X being in the equatorial plane through the Greenwich meridian. The two rotations account for the longitude of the orbit plane at t_k and the inclination of the orbit with respect to the earth's equatorial plane.

Figures 37 through 39 clearly show how the Transit orbit parameters are interpreted and how they are used to obtain a definition of the that the transmitted frequency is offset low by about 80 ppm (32 kHz at 400 MHz) to prevent f_R from crossing 400 MHz.

The navigation receiver is equipped with a stable reference oscillator from which a 400 MHz ground reference frequency f_G is derived. Oscillator stability must be adequate to assume a constant frequency throughout the satellite pass. As shown by the figure, the navigation receiver forms the difference frequency f_G-f_R, and each Doppler measurement is a count of the number of difference frequency cycles occuring between time marks received from the satellite. Because every message bit effectively represents another time mark, the Doppler counting intervals are formed with respect to the message format of Figure 35. For example, each line of the message lasts about 4.6 seconds, and the commonly used Doppler count interval of 23 seconds is formed by starting a new count at the end of every fifth line.

Each Doppler count is composed of two parts: the count of a constant difference frequency f_G - f_T, minus the count of the number of Doppler cycles received during that time interval. It is the Doppler cycle count which is physically meaningful. The count of the difference frequency is an additive constant which is eliminated by the position fix calculation.

Figure 40 emphasizes that the distance between the satellite and the observer changes throughout the satellite pass. It is this change, in fact, which causes the Doppler frequency shift. As the satellite moves closer, more cycles per second must be received than were transmitted to account for the shrinking number of wavelengths along the propagation path. For each wavelength the satellite moves closer, one additional cycle must be received. Therefore, the Doppler frequency count is a direct measure of the change in distance between the receiver and the satellite over the Doppler count interval. In other words, the Doppler count is a geometric measure of the range difference between the observer and the satellite at two points in space, accurately defined by the navigation message.

This is a very sensitive measure because each count represents one wavelength, which at 400 MHz is only 0.75 meter.

The equation defining the Doppler count of f_G-f_R is the integral of this difference frequency over the time interval between receipt of time marks from the satellite. For example,

$$N_1 = \int_{t_1 + R_1/C}^{t_2 + R_2/C} (f_G - f_R)\, dt \qquad (1)$$

Note that $t_1 + R_1/C$ is the time of receipt of the satellite time mark that was transmitted at time t_1. The signal is received after propagating over distance R_1 at the velocity of light C.

Equation 1 represents the actual measurement made by the satellite receiver, but it is helpful to expand this equation into two parts:

$$N_1 = \int_{t_1 + R_1/C}^{t_2 + R_2/C} f_G\, dt - \int_{t_1 + R_1/C}^{t_2 + R_2/C} f_R\, dt \qquad (2)$$

Because the first integral in Equation 2 is of a constant frequency f_G, it is easy to integrate, but the second integral is of the changing frequency f_R. However, the second integral represents the number of cycles received between the times of receipt of two timing marks.

By a "conservation of cycles" argument, this quantity must equal identically the number of cycles transmitted during the time interval between transmission of these time marks. Using this identity, Equation 2 can be written

$$N_1 = \int_{t_1 + R_1/C}^{t_2 + R_2/C} f_G \, dt - \int_{t_1}^{t_2} f_T \, dt \tag{3}$$

Because the frequencies f_G and f_T are assumed constant during a satellite pass, the integrals in Equation 3 become trivial, resulting in

$$N_1 = f_G \left[(t_2 - t_1) + \frac{1}{C} (R_2 - R_1) \right] - f_T (t_2 - t_1) \tag{4}$$

Rearranging the terms in Equation 4 gives

$$N_1 = (f_G - f_T)(t_2 - t_1) + \frac{f_G}{C} (R_2 - R_1) \tag{5}$$

Equation 5 clearly shows the two parts of the Doppler count. First is the constant difference frequency multiplied by a time interval defined by the satellite clock. Second is the direct measure of slant range change measured in wavelengths of the ground reference frequency C/f_G. It happens that the wavelength of f_G is the proper scale factor because received time marks are used to start and stop the Doppler counts. If a ground clock is used to control the count intervals, the wavelength of f_T would become the appropriate scale factor.

5.4 COMPUTING THE FIX

A usable satellite pass will be above the horizon between 10 and 18 minutes, which determines the number of Doppler counts acquired. Typically 20 to 40 counts will be collected by modern equipment. The Doppler counts and the satellite navigation message are passed to a small digital computer for processing. For simplicity, we will assume a stationary receiver as shown in Figure 40 in order to establish the basic position fix concept.

The computer first takes advantage of message redundancy to eliminate errors in the received orbit parameters. It is then able to compute the satellite's position at the beginning and end of every Doppler count. The computer also receives an estimate of the navigator's position in three dimensions, i.e., latitude, longitude, and altitude above the reference ellipsoid. The equations of Figure 41 are used to convert the navigator's position into the same Cartesian coordinate system shown in Figure 39, which permits the slant range from the navigator to each satellite position to be calculated. It is then possible to compare the slant range change measured by each Doppler count with the corresponding value computed from the estimated navigator's position.

The difference between a Doppler measured slant range change and the value computed from the estimated position is called a residual e_i. The objective of the position fix calculation is to find the navigator's position which minimizes the sum of the squares of the residuals (i.e., makes the calculated slant range change values agree best with the measured values). To implement the solution, a simple, linear estimate is made of the effect each variable will have on each residual. Assuming we wish to solve for latitude (ϕ), longitude (λ), and the unknown frequency offset $F = f_G - f_T$, we can write

$$\hat{e}_i = e_i - \frac{\partial e_i}{\partial \phi} \Delta\phi - \frac{\partial e_i}{\partial \lambda} \Delta\lambda - \frac{\partial e_i}{\partial F} \Delta F \qquad (6)$$

This equation states that if we move the estimated position by $\Delta\phi$ and by $\Delta\lambda$ and the estimated frequency offset by ΔF, the present residual e_i will become a new value, estimated to be $\hat{e}_i$. Next we wish to minimize the sum of the squares of the estimated residuals by setting the partial derivative with respect to each variable equal to zero. This results in three equations, where the summation covers the m valid Doppler count residuals.

$$\frac{\partial}{\partial\phi} \sum_{i=1}^{m} \hat{e}_i^2 = 2 \sum_{i=1}^{m} \left(\hat{e}_i \cdot \frac{\partial \hat{e}_i}{\partial \phi} \right) = 0$$

$$\frac{\partial}{\partial\lambda} \sum_{i=1}^{m} \hat{e}_i^2 = 2 \sum_{i=1}^{m} \left(\hat{e}_i \cdot \frac{\partial \hat{e}_i}{\partial \lambda} \right) = 0 \qquad (7)$$

$$\frac{\partial}{\partial F} \sum_{i=1}^{m} \hat{e}_i^2 = 2 \sum_{i=1}^{m} \left(\hat{e}_i \cdot \frac{\partial \hat{e}_i}{\partial F} \right) = 0$$

Ignoring all but the first-order terms of Equations 7 gives three equations in the three selected variables, $\Delta\phi$, $\Delta\lambda$, and ΔF.

$$\sum_{i=1}^{m} \frac{\partial e_i}{\partial \phi} \left[e_i - \frac{\partial e_i}{\partial \phi} \Delta\phi - \frac{\partial e_i}{\partial \lambda} \Delta\lambda - \frac{\partial e_i}{\partial F} \Delta F \right] = 0$$

$$\sum_{i=1}^{m} \frac{\partial e_i}{\partial \lambda} \left[e_i \quad \frac{\partial e_i}{\partial \phi} \Delta\phi - \frac{\partial e_i}{\partial \lambda} \Delta\lambda - \frac{\partial e_i}{\partial F} \Delta F \right] = 0 \qquad (8)$$

$$\sum_{i=1}^{m} \frac{\partial e_i}{\partial F} \left[e_i - \frac{\partial e_i}{\partial \phi} \Delta\phi - \frac{\partial e_i}{\partial \lambda} \Delta\lambda - \frac{\partial e_i}{\partial F} \Delta F \right] = 0$$

		WGS-72 VALUES
ϕ	= LATITUDE	
λ	= LONGITUDE	
H	= HEIGHT ABOVE ELLIPSOID	
a	= SEMI-MAJOR AXIS	(6378135 METERS)
f	= FLATTENING COEFFICIENT	(1/298.26)
b	= $a(1-f)$ = SEMI-MINOR AXIS	(6356750.52 METERS)
e	= $\sqrt{f(2-f)}$ = ECCENTRICITY	
RN	= RADIUS OF CURVATURE IN THE PRIME VERTICAL	
RN	= $a/(1-e^2 \, \mathrm{SIN}^2\phi)^{1/2}$	
XN	= $(RN + H) \, \mathrm{COS}\, \phi \, \mathrm{COS}\, \lambda$	
YN	= $(RN + H) \, \mathrm{COS}\, \phi \, \mathrm{SIN}\, \lambda$	
ZN	= $[RN\,(1-e^2) + H] \, \mathrm{SIN}\, \phi$	

Figure 41. Relating Latitude and Longitude to Cartesian Coordinates

Because only linear, first-order terms are used, the values of $\Delta\phi$, $\Delta\lambda$, and ΔF which satisfy these equations will be an approximation to the exact solution. Therefore, the original estimates of latitude, longitude, and frequency are adjusted in accordance with the first solution, and new slant ranges, residuals, and partial derivatives of the residuals are computed for another solution. This process is repeated, or iterated, until the computed values of $\Delta\Phi$, $\Delta\lambda$, and ΔF are sufficiently small, at which point the solution is said to have converged. Normally, only two or three iterations are required, even when the initial estimate is tens of kilometers from the final solution. Note that ignoring higher order terms has no effect on final accuracy, because these terms tend to zero as the solution converges.

In summary, the Transit position fix begins with an estimated position and determines the shift in that position required to best match calculated slant range differences with those measured by the Doppler counts. The initial estimate can be in error by 200 or 300 km and the solution will converge to an accurate value.

5.5 ACCOUNTING FOR MOTION

If the navigator is in motion during the satellite pass, the motion must be recorded before an accurate position fix can be computed. As Figure 5 in Chapter 2 shows, only if the motion is known can the calculated range differences from satellite to receiver be compared properly with the range differences measured by the Doppler counts. Automatic speed and heading inputs often are employed for this purpose. During the satellite pass the computer creates a table of the navigator's estimated latitude and longitude at the beginning and end of each Doppler count interval. As before, the fix solution provides a delta-latitude and a delta-longitude, which are added to every point in the navigator's table between iterations of the solution. Therefore, although the final position fix result may be expressed as a latitude and a longitude at one point in time, the fix solution in fact is a shift of the entire estimated track.

CHAPTER 6
ACCURACY CONSIDERATIONS

6.1 STATIC SYSTEM ERRORS

Reference 11 presents an error budget for individual Transit position fixes that provides a good summary of the factors affecting accuracy when the navigator is not moving:

Source		Error (meters)
1.	Uncorrected propagation effects (ionospheric and tropospheric effects)	1-5
2.	Instrumentation and measurement noise (local and satellite oscillator phase jitter, navigator's clock error)	3-6
3.	Uncertainties in the geopotential model used in generating the orbit	10-20
4.	Uncertainties in navigator's altitude (generally results in bias in longitude)	10
5.	Unmodeled polar motion and UT1-UTC effects	0-10
6.	Incorrectly modeled surface forces (drag and radiation pressure acting on the satellites during extrapolation interval)	10-25
7.	Ephemeris rounding error (last digit in ephemeris is rounded)	5

Since publication of this table in 1973, the polar motion error has been modeled and is included as an adjustment to the transmitted orbit parameters. The root sum square (rss) of the remaining errors lies in the range of 18 to 35 meters, which we believe is slightly optimistic due to the laboratory standards and the sophisticated refraction correction models employed by the Applied Physics Laboratory. Field results usually lie in the range of 27 to 37 meters rss. Figure 6 presented a typical set of stationary fix results. The

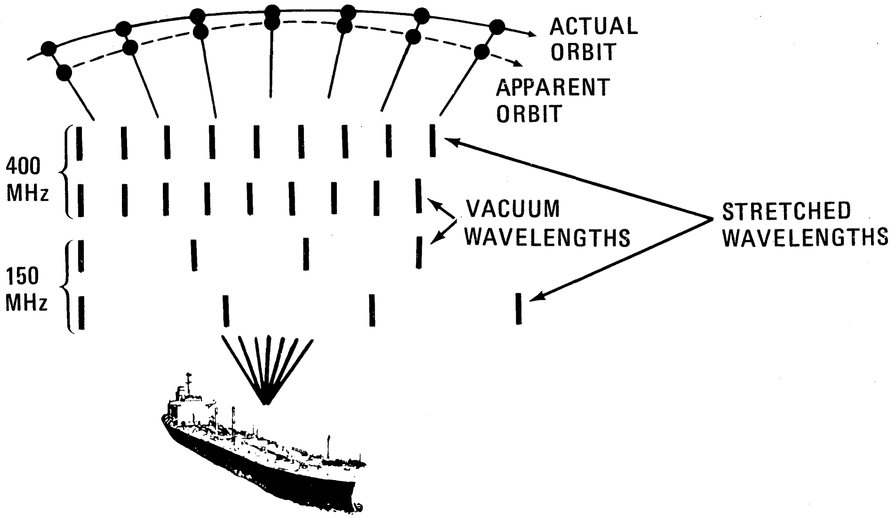


Figure 42. Ionospheric Refraction Stretches Signal Wavelength Causing Greater Apparent Orbit Curvature

maximum error was 77 meters, and the rms radial error was 32 meters for all 69 points.

6.1.1 Refraction Errors

There are two sources of refraction error; the larger one is due to the ionosphere. As illustrated by Figure 42, as the 150 and 400 MHz signals pass through the ionosphere, their wavelengths are stretched because of interaction with free electrons and ions. This stretching represents a phase velocity greater than the speed of light, which is characteristic of a dispersive medium. To a close first-order approximation, the wavelength stretch is inversely proportional to the square of transmitted frequency. Because satellite motion changes the path length through the ionosphere, the rate of change of this stretch causes an ionospheric refraction error frequency shift in the received signal. Reference 3 showed that an excellent refraction correction could be obtained by combining the Doppler measurements made at two different frequencies, and this is why Transit satellites transmit both 150 and 400 MHz signals.

For applications not requiring the ultimate system accuracy,

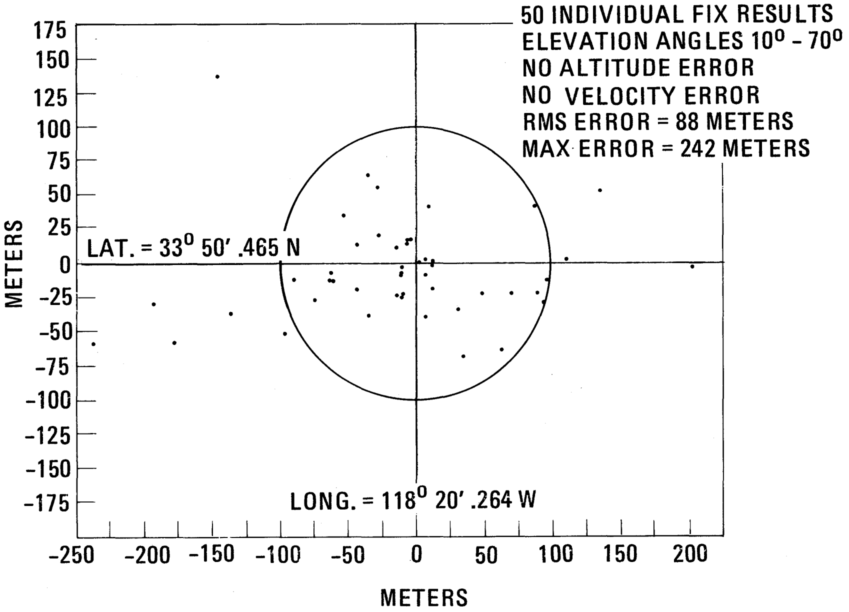


Figure 43. Typical Single-Channel Transit Position Fix Results

400 MHz signal-channel receiving equipment can be used. Figure 42 demonstrates that because of wavelength stretching, the satellite will appear to follow a path with greater curvature about the navigator. The effect is to reduce the total Doppler shift somewhat, pushing the position fix solution away from the satellite orbit to explain the lower Doppler slope. Because the satellites move primarily along north-south lines, the resultant navigation errors are mostly in longitude. The magnitude of these errors varies with density of the ionosphere from very small at night to peaks of 200 to 500 meters in daylight, depending on sunspot activity and location with respect to the magnetic equator where the ionosphere is most dense. Figure 43 is a plot of typical single-channel results containing both daytime and nighttime fixes in which the maximum error is 242 meters and the rms error is 88 meters.

The second source of refraction error is the troposphere. In this case, propagation speed is slowed as the signal passes through the

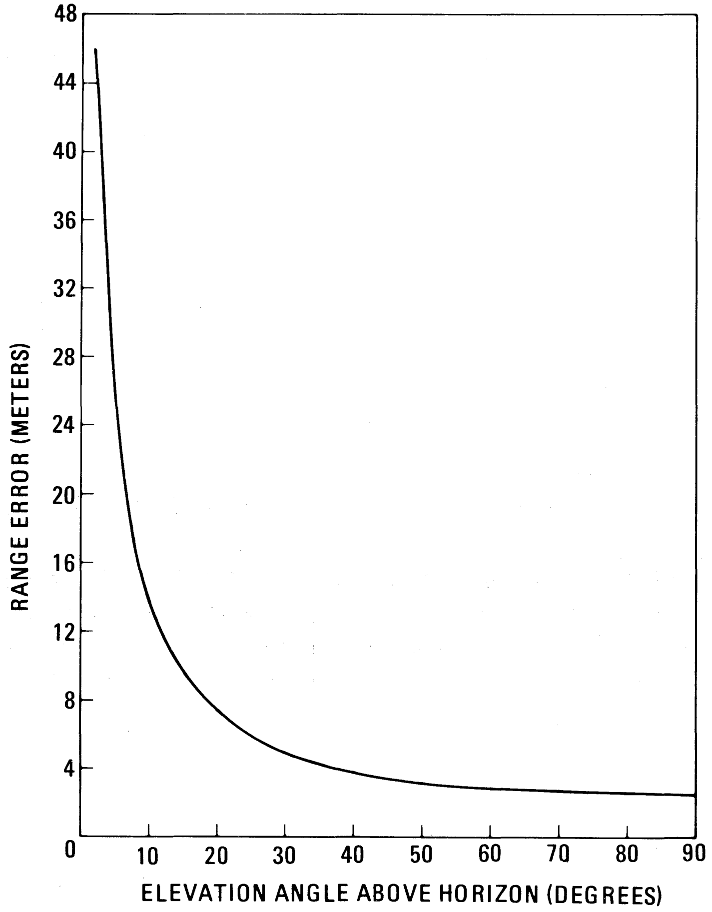


Figure 44. Typical Range Measurement Error Due to Trospheric Refraction

earth's atmosphere, which compresses the signal wavelength. The effect is directly proportional to transmitted frequency, as is the Doppler shift, and therefore it cannot be detected like ionospheric refraction. There are only two ways to reduce the effect of tropospheric refraction. First is by modeling its effect on the Doppler counts. Very sophisticated models employing measurements of temperature, pressure, and humidity have been published for this purpose, but less sophisticated models are usually sufficient (Reference 8). This is especially true in conjunction with the second

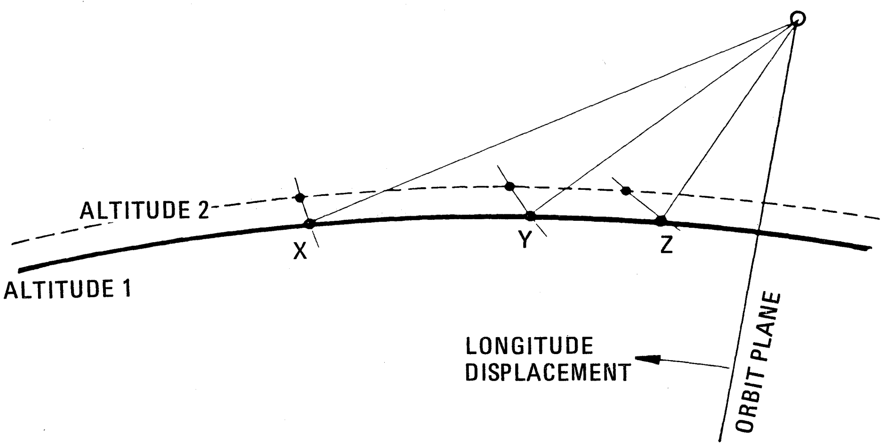


Figure 45. Effect of Altitude Estimate on Position Fix

technique, which is to delete Doppler data taken close to the horizon where the tropospheric refraction error is greatest. Above 5^o to 10^o of elevation, the tropospheric error is many times smaller than at the horizon, as illustrated in Figure 44 which shows typical magnitude of range error as a function of elevation above the horizon.

6.1.2 Altitude Error

The specific Doppler curve obtained as a satellite passes is predominantly a function of the navigator's position along the line of satellite motion and his distance from the orbit plane. Because Transit satellites are in polar orbits, the along-track position closely relates to latitude and the cross-track distance is a combination of longitude and altitude.

Figure 45 is the cross section of a pass where the satellite is moving in its orbital plane perpendicular to the page. It has just reached the center of pass with respect to stations X, Y, and Z. The figure illustrates how the cross-track distance is a function of both longitude and altitude, which affect the Doppler curve in similar ways. To compute an accurate fix, therefore, it is necessary to have *a priori* knowledge of altitude. Figure 46 shows the sensitivity of fix error to altitude error as a function of maximum satellite pass elevation

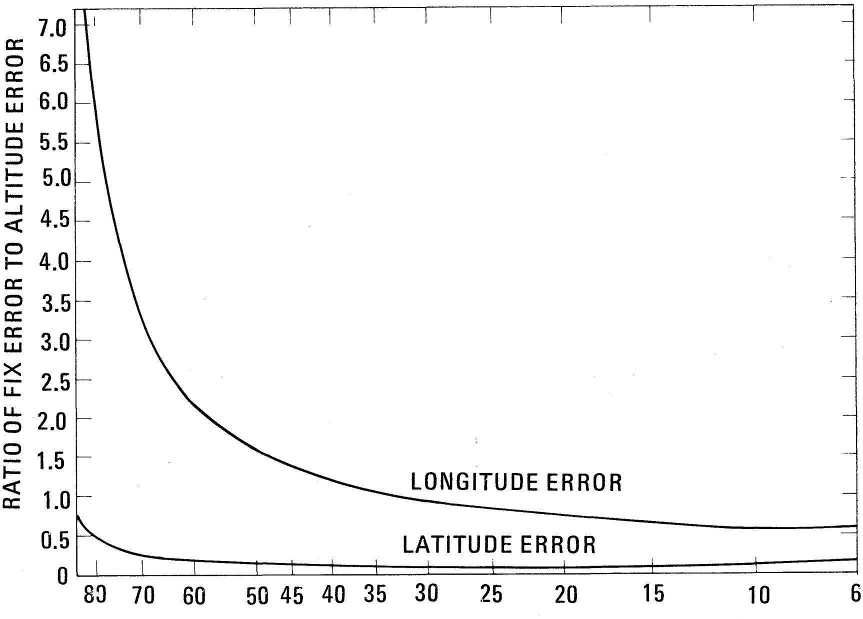
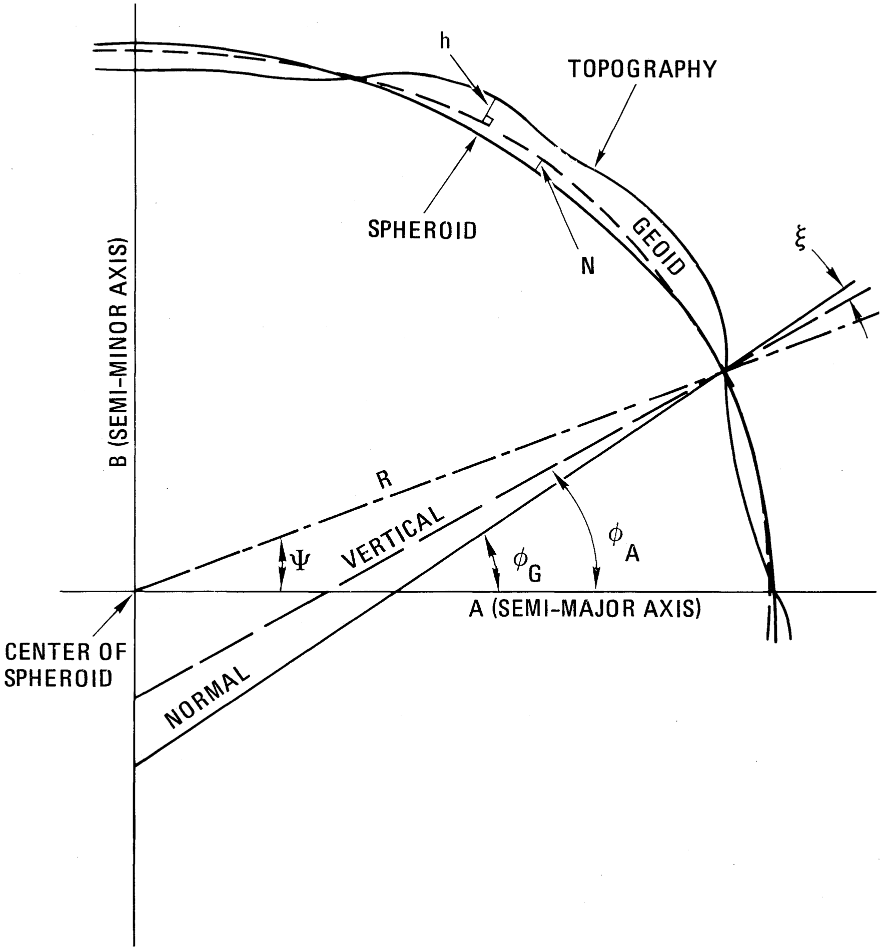


Figure 46. Sensitivity of Satellite Fix to Altitude Estimate Error

angle. The elevation angle is plotted on a scale that is uniform in probability of satellite pass occurrence. In other words, more passes fall between 10° and 20° than between 70° and 80°, except at very high latitudes.

For satellite navigation "altitude" means height above or below the reference spheroid (the reference ellipsoid or satellite datum). This surface is chosen to be a worldwide best fit to mean sea level, which is the true geoid. Figure 47 illustrates the differences between the geoid, the spheroid, and topography. Therefore, knowing height above mean sea level is not sufficient for an accurate position fix. One also must know the local geoidal height, which is the deviation between the geoid and the spheroid. Figure 48 is a geoidal height map indicating that these deviations reach nearly 100 meters.



Ψ = GEOCENTRIC LATITUDE

R = GEOCENTRIC RADIUS

h = ELEVATION ABOVE GEOID

N = GEOID HEIGHT

ϕ_A = ASTRONOMIC LATITUDE

ϕ_G = GEODETIC LATITUDE

ξ = DEFLECTION OF THE VERTICAL

Figure 47. Relationships of Geodetic Surfaces (From NASA Directory of Observation Station Locations, 2nd Ed., Vol. 1, Nov. 1971, Goddard Space Flight Center)

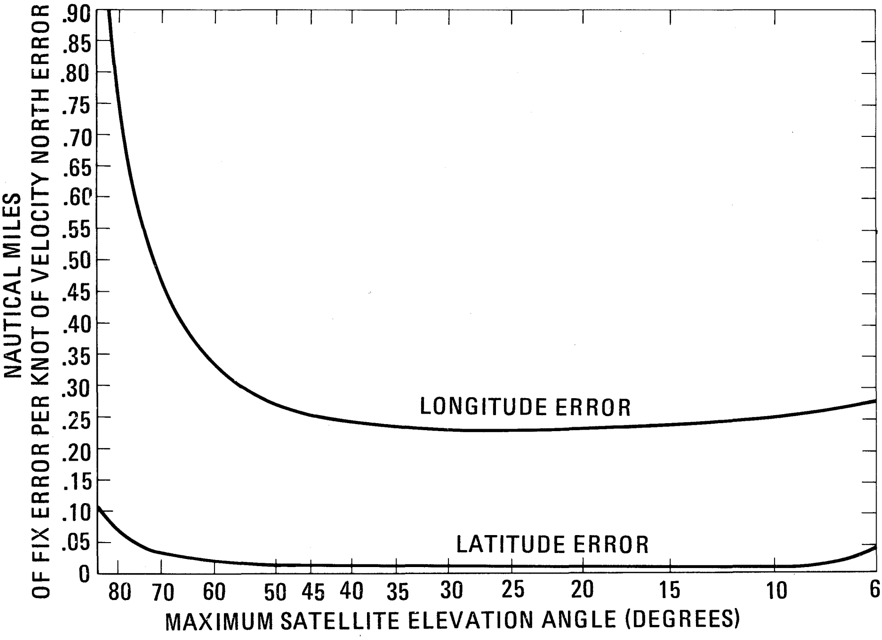


Figure 50. Sensitivity of Satellite Fix to a One-Knot Velocity North Estimate Error

indicate realistic rms performance levels. One can see that a dual-channel system provides maximum benefit when there is an accurate source of velocity. The other benefit of the dual-channel system is to eliminate the peak 200 to 500 meter errors which occur with single-channel equipment during the day, dependent on sunspot activity.

6.3 VELOCITY SOLUTION

The normal position fix solution determines latitude, longitude, and frequency offset by means of Equations 8. These equations easily could be expanded to include other system variables such as velocity north, velocity east, altitude, and even acceleration. With every new variable, however, accuracy would become more and more sensitive to system noise. In fact, studies have shown that velocity north is the only parameter which can be added without

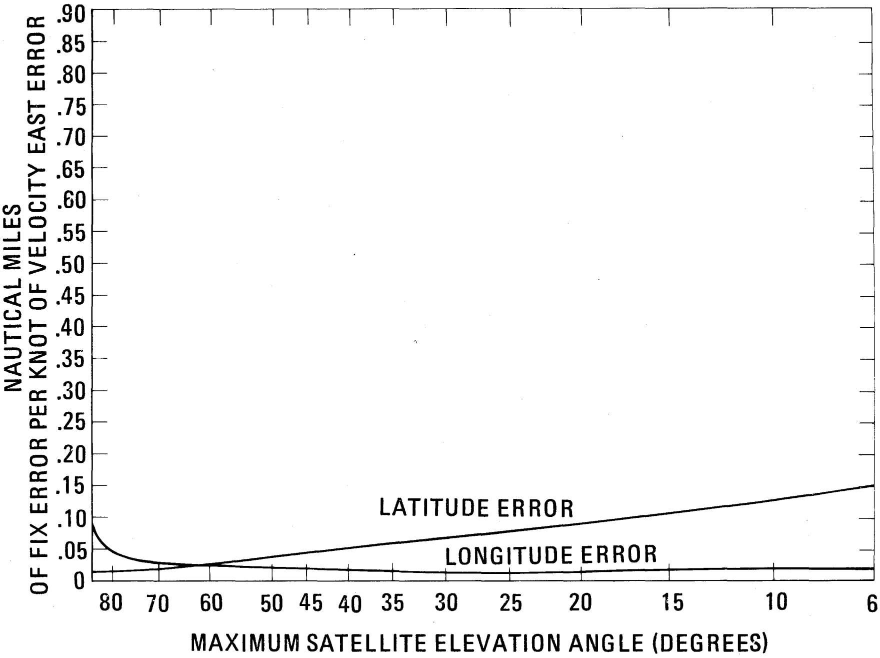


Figure 51. Sensitivity of Satellite Fix to a One-Knot Velocity East Estimate Error

creating intolerable noise sensitivity; that is, it is the only other variable which affects Doppler curve shape in a way that can be discerned clearly from the effects of latitude, longitude, or frequency. To be precise, the added variable should be velocity parallel to satellite motion, but velocity north is an adequate approximation at most latitudes because the satellites are in polar orbits.

Solving for velocity north increases position fix error when ship's motion is accurately known. Therefore it should be attempted only when velocity errors are likely to exceed about 0.4 knot. The expanded solution is more sensitive to other sources of system noise, such as asymmetric Doppler data, and it does not work well for pass elevation angles below 20°. Finally, the velocity north result becomes the scapegoat for other system errors and is not a dependable measure of velocity north error; it simply allows the latitude and longitude to be more accurate in the face of large velocity errors.

6.4 REFERENCE DATUM

It is important to realize that maps are drawn and positions are defined with respect to a reference datum. In the United States we use the North American Datum, in Japan the Tokyo Datum, in Europe the European Datum, etc. The Transit system currently uses the World Geodetic System of 1972 (WGS-72). As a result, the same reference marker will have a different set of latitude and longitude coordinates in each reference datum. Apparent differences of 1/2-kilometer occur in some locations.

The four parts of Figure 52 help us visualize the concept of reference datums and how they relate to each other. Figures 47 and 48 already indicated that the earth is an irregular shape due to density (gravity) variations, and Figure 52(a) is an exaggerated model of an irregular "earth". The surface shown represents the geoid, which is defined as the location of mean sea level over the entire earth's surface.

In order to make reasonably accurate maps, a model of the earth's surface is needed. Figure 52(b) shows how such models have been designed to fit the earth over the area of local interest, which in the past never was larger than a continent. The model consists of a spheroid (ellipsoid) and one position called the datum at which latitude and longitude are defined. Such a model works well and allows accurate maps to be drawn in the vicinity of the datum.

Now that satellites are being used to measure the geoid (satellite geodesy), a different type of datum is needed. As illustrated by Figure 52(c), a world spheroid may not fit the earth very well at any one location, but it is a "best fit" to the entire earth. In addition, there is not a single reference datum position because many satellite tracking stations are involved, and their positions are defined as part of the calculations which determine the earth's geopotential field (geoid). The WGS-72 spheroid is a "best fit" to the WGS-72 geoid.

Figure 52(d) makes it clear that there must be some method of relating a position in one datum to coordinates in another. For

(A) THE EARTH DOES NOT HAVE A UNIFORM SURFACE

(B) LOCAL DATUMS HAVE BEEN DEVELOPED TO FIT LIMITED AREAS OF INTEREST

(C) SATELLITE NAVIGATION REQUIRES A DATUM WHICH IS A "BEST FIT" TO THE ENTIRE EARTH

(D) A DATUM SHIFT CALCULATION IS REQUIRED WHEN TRANSFERRING COORDINATES FROM ONE SYSTEM TO ANOTHER

Figure 52. Development and Relationship of Local and Global Reference Datums

example, satellite position fixes taken in Tokyo harbor might show the ship to be well inland when plotted on a local chart. The reason is datum difference as illustrated by Figure 52(d).

The coordinate differences between two datums can be resolved by knowledge of three (or four) offset parameters and the size and shape of each spheroid. First is the Δx, Δy, and Δz offset between the center of the two spheroids. Sometimes a longitude rotation is needed as a fourth offset. The size and shape of each spheroid are defined by the semi-major axis (equatorial radius) and by the flattening coefficient.

Reference 13 lists datum shift constants which can be used in converting from various datums to WGS-72, shown here in Figure 53. Caution should be exercised in trusting the results for two reasons. First is that Reference 13 indicates the accuracy of each offset constant is only ±5 meters in North America, ±10 meters in Europe, and ±15 meters in Japan and Australia. Part of this uncertainty is due to distortions in the local reference datum. The second reason is that the offset parameters were determined empirically with Geoceiver surveys using precise ephemeris orbits (see Section 3.5.4). Unfortunately, there are differences of perhaps 10 meters between positions determined with precise ephemeris orbits from the Defense

DATUM	SPHEROID	SEMI-MAJOR AXIS	RECIPROCAL FLATTENING	SHIFT TO WGS-72 a = 6378135 1/f = 298.26		
		METERS		METERS		
				ΔX	ΔY	ΔZ
NAD 1927	CLARKE 1866	6378206	294.98	-22*	157*	176*
EUROPEAN	INTERNATIONAL	6378388	297.00	-84	-103	-127
TOKYO	BESSEL	6377397	299.15	-140	516	673
AUSTRALIAN NATIONAL	REFERENCE ELLIPSOID 1967	6378160	298.25	-122	-41	146
OLD HAWAIIAN	CLARKE 1866	6378206	294.98			
MAUI				65	-272	-197
OAHU				56	-268	-187
KAUAI				46	-271	-181
CAPE (ARC)	CLARK 1880 (MOD)	6378249	293.47	-129	-131	-282
SOUTH AMERICAN	REFERENCE ELLIPSOID 1967	6378160	298.25	-77	3	-45
ORDNANCE SURVEY OF GREAT BRITAIN 1936	AIRY	6377563	299.32	368	-120	425
JOHNSTON ISLAND ASTRO 1961	INTERNATIONAL	6378388	297.00	192	-59	-211
WAKE-ENIWETOK 1960	HOUGH	6378270	297.00			
KWAJALEIN ATOLL				112	68	-44
WAKE ISLAND				121	62	-22
ENIWETOK ATOLL				144	62	-38
WAKE ISLAND ASTRO 1952	INTERNATIONAL	6378388	297.00	283	-44	141
CANTON ISLAND ASTRO 1966	INTERNATIONAL	6378388	297.00	294	-288	-382
GUAM 1963	CLARKE 1866	6378206	294.98	-89	-235	254
ASCENSION ISLAND ASTRO 1958	INTERNATIONAL	6378388	297.00	-214	91	48
SOUTH ASIA	FISCHER 1960	6378155	298.30	21	-61	-15
NANKING 1960	INTERNATIONAL	6378388	297.00	-131	-347	0
ADINDAN	CLARKE 1880	6378249	293.47	-152	-26	212
MERCURY 1960	FISCHER 1960	6378155	298.30			
NAD 27 AREA				-25	46	-49
ED AREA				-13	-88	-5
TD AREA				18	-132	60
MODIFIED MERCURY 1968	FISCHER 1968	6378150	298.30			
NAD 27 AREA				-4	12	-7
ED AREA				-3	1	-6
TD AREA				22	34	2

*VALUES OF -9, 139, AND 173 SHOULD BE USED FOR ALASKA AND CANADA

Figure 53. Datum Shift Constants

Mapping Agency and those determined with orbits transmitted from the Transit satellites. Figure 54, from Reference 13, gives the Molodensky formulas most often used to transform coordinates from one reference system to another.

A. THE STANDARD MOLODENSKY FORMULAS

$$\begin{aligned}\Delta\phi'' = \Big\{ & -\Delta X \sin\phi \cos\lambda - \Delta Y \sin\phi \sin\lambda + \Delta Z \cos\phi \\ & + \Delta a \,(R_N e^2 \sin\phi \cos\phi)/a \\ & + \Delta f \,[R_M (a/b) + R_N (b/a)] \sin\phi \cos\phi \Big\} \cdot [(R_M + H) \sin 1'']^{-1}\end{aligned}$$

$$\Delta\lambda'' = [-\Delta X \sin\lambda + \Delta Y \cos\lambda] \cdot [(R_N + H) \cos\phi \sin 1'']^{-1}$$

$$\begin{aligned}\Delta H = {} & \Delta X \cos\phi \cos\lambda + \Delta Y \cos\phi \sin\lambda + \Delta Z \sin\phi \\ & - \Delta a \,(a/R_N) + \Delta f \,(b/a)\, R_N \sin^2\phi\end{aligned}$$

B. THE ABRIDGED MOLODENSKY FORMULAS

$$\begin{aligned}\Delta\phi'' = {} & [-\Delta X \sin\phi \cos\lambda - \Delta Y \sin\phi \sin\lambda + \Delta Z \cos\phi + (a\Delta f + f\Delta a) \sin 2\phi] \\ & \cdot [R_M \sin 1'']^{-1}\end{aligned}$$

$$\Delta\lambda'' = [-\Delta X \sin\lambda + \Delta Y \cos\lambda] \cdot [R_N \cos\phi \sin 1'']^{-1}$$

$$\Delta H = \Delta X \cos\phi \cos\lambda + \Delta Y \cos\phi \sin\lambda + \Delta Z \sin\phi + (a\Delta f + f\Delta a) \sin^2\phi - \Delta a$$

C. DEFINITION OF TERMS IN THE MOLODENSKY FORMULAS

ϕ, λ, H = GEODETIC COORDINATES (OLD ELLIPSOID)

ϕ = GEODETIC LATITUDE. THE ANGLE BETWEEN THE EARTH'S EQUATORIAL PLANE AND THE ELLIPSOIDAL NORMAL AT A POINT (MEASURED POSITIVE NORTH FROM THE EQUATOR, NEGATIVE SOUTH).

λ = GEODETIC LONGITUDE. THE ANGLE BETWEEN THE PLANE OF THE GREENWICH MERIDIAN AND THE PLANE OF THE GEODETIC MERIDIAN OF THE POINT (MEASURED IN THE PLANE OF THE EQUATOR, POSITIVE EAST FROM GREENWICH).

H = THE DISTANCE OF A POINT FROM THE ELLIPSOID MEASURED ALONG THE ELLIPSOIDAL NORMAL THROUGH THE POINT.

$H = N + \overset{*}{h}$

N = GEOID-ELLIPSOID SEPARATION. THE DISTANCE OF THE GEOID ABOVE (+N) OR BELOW (-N) THE ELLIPSOID.

$\overset{*}{h}$ = DISTANCE OF A POINT FROM THE GEOID (ELEVATION ABOVE OR BELOW MEAN SEA LEVEL).

$\Delta\phi, \Delta\lambda, \Delta H$ = CORRECTIONS TO TRANSFORM THE GEODETIC COORDINATES FROM THE OLD DATUM TO WGS.

$\Delta X, \Delta Y, \Delta Z$ = SHIFTS BETWEEN ELLIPSOID CENTERS OF THE OLD DATUM AND WGS.

a = SEMIMAJOR AXIS OF THE OLD ELLIPSOID.

*b = SEMIMINOR AXIS OF THE OLD ELLIPSOID.

*$b/a = 1 - f$

f = FLATTENING OF THE OLD ELLIPSOID.

$\Delta a, \Delta f$ = DIFFERENCES BETWEEN THE PARAMETERS OF THE OLD ELLIPSOID AND THE WGS ELLIPSOID (WGS MINUS OLD).

e = ECCENTRICITY.

$e^2 = 2f - f^2$

R_N = RADIUS OF CURVATURE IN THE PRIME VERTICAL

$R_N = a/(1-e^2 \sin^2 \phi)^{1/2}$

R_M = RADIUS OF CURVATURE IN THE MERIDIAN.

$R_M = a(1-e^2)/(1-e^2 \sin^2 \phi)^{3/2}$

NOTE: ALL Δ-QUANTITIES ARE FORMED BY SUBTRACTING OLD ELLIPSOID VALUES FROM WGS ELLIPSOID VALUES.

*INDICATES PARAMETERS WHICH DO NOT APPEAR IN THE ABRIDGED MOLODENSKY FORMULAS.

Figure 54. Datum Shift Equations (From References 8 and 13)

CHAPTER 7
CONCLUSION

This document has provided an in-depth review of the Transit system from the user's point of view. Except for a classified Soviet system, Transit is the only navigation satellite system available today. Furthermore, because of propagation limitations of the Omega system, Transit is the only system which provides truly worldwide coverage. This situation will continue until at least 1985, or later, when NAVSTAR, the Global Positioning System, is expected to become operational. As proposed by the Office of Telecommunications Policy (Reference 12), a ten-year overlap period from the time NAVSTAR becomes operational will allow users to depreciate Transit equipment before having to purchase NAVSTAR equipment. The ten-year overlap also will give time for NAVSTAR manufacturers to develop, improve, and produce a sufficient range of equipment to serve the many expected applications (Reference 23). Thus, we feel certain that Transit will continue to provide its most useful service until at least 1995.

We have shown that Transit is an extremely reliable system in delivering accurate position fixes to its users. The reliability is based on many factors. Signals are provided on a direct, line-of-sight basis from the satellite to the user, avoiding the propagation problems that plague earth-based transmitters. The Navy Astronautics Group has established a remarkable record for maintaining a reliable message in each satellite memory. The satellites themselves are extremely reliable, with three which are operating extremely well after more than ten years of service. The twelve spacecraft in storage assure that the system can be maintained in service for many years, even when the present satellites cease to function.

We have looked at the amazing breadth of Transit system applications, ranging from use aboard fishing boats to military submarines. If the user population growth trend continues, there will be more than 10,000 Transit system users by the early 1980's. Complementing the growth in applications and in the number of users is development of a new generation of Transit satellite called NOVA. Thus, there are many signs that the system is growing and fulfilling vital needs around the world.

Finally, this document has described both the theory of Transit satellite navigation and the factors which affect accuracy performance. This has included a definition of the orbit message parameters, the meaning of the Doppler counts, and a review of the position fix concept. The inherent system accuracy was described, and sensitivity curves were given for external factors which affect position fix accuracy.

The primary objective of this document has been to provide an extensive and detailed review of the Transit system today. A fascinating story has emerged. The system was developed almost exclusively to guide Polaris submarines, and it continues to serve this purpose extremely well. However, the U.S. Government also released the system for commercial use, and on their own initiative manufacturers around the world began to produce Transit navigation equipment. A wide variety of users are now experiencing the advantages of accurate, worldwide, all-weather navigation. The momentum of use continues to build, and Transit is destined to play a vital role in the world navigation scene for another decade or two.

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